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Mathematical substantiation of diagnostics of the technical condition of the oil pipeline

Abstract: The paper is proposed a mathematical model of source recovery for the partial differential equation of parabolic type on a tree graph. The developed model makes it possible to apply the results in the diagnostics of the operation of large-scale pipelines through which liquids are pumped, in particular, the detection of a blockage in a separate link of the main oil pipeline.

Keywords: main oil pipeline, blockage in a link of an oil pipeline, heat equation, tree-graph, source identification, boundary control method, leaf peeling method.

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1. Introduction

Operation of a main oil pipeline presupposes the activities necessary for the continuous, proper and efficient functioning of the oil pipeline, including diagnostics and operational dispatch control.

The mathematical model is reduced to the inverse problem for the heat equation on a tree-graph. According to the model, diagnostics of the linear part of the main oil pipeline is carried out by installing a sensor at any point (vertex of the graph) of the main oil pipeline of one region this makes it possible to find out the state on any fragment of the linear part (edge of the graph) in another region where the dispatch center is located.

Diagnostics of the linear part of main oil pipelines is carried out to ensure safety, detect and classify defects (failures) and their precise localization, determine the possibility of their further operation at design technological modes, and calculate the permissible operating pressure [1].

Based on the results of technical diagnostics, a conclusion is drawn up on the technical condition of the equipment, taking into account the identification of incidents, which will prevent the risks of failure and monitor the technical condition of the main oil pipeline.

An incident implies a failure or damage to a production facility, as well as a deviation from the mode of the technological process. It is usually presented in the form of damage and congestion of the linear part.

Damage to the linear part is characterized by a violation of integrity, leading to loss of oil in the pipeline. Congestion is characterized by the accumulation of impurities and impurities on the inner side of the pipeline walls, leading to a decrease in oil flow.

Practical application of the developed model will make it possible to identify these types of incidents along the entire length of the main oil pipeline for their further elimination.

Controllability and inverse problems for parabolic equations on a tree-graph are related to the cable equation. From a practical point of view, such models find application in studying the influence of a continuous medium of a fluid flow through a main network. We consider the theoretical description from the point of view of source recovery in the cable equation.

2. Mathematical formulation of the problem

Denote by $\Omega = \{E, V\}$ a finite connected compact metric tree graph, where $E = \{e_1, e_2, \dots, e_N\}$ is the set of edges and $V = \{\nu_1, \nu_1, \dots \nu_{N+1}\}$ a set of vertices. For a metric graph, each edge $e_j \in E$ is identified with the interval (a_{2j-1}, a_{2j}) of a real line of positive length $l_j = \lfloor a_{2j-1} - a_{2j} \rfloor$. A tree graph has no cycles. The edges of the graph converge at the vertices ν_j, which create the endpoint equivalence class $\{a_j\}$. The star graph consists of all edges included in one internal vertex ν. A pencil-graph is a star graph, all of whose edges, except one, are boundary edges of the graph Ω.

We consider a connected finite compact metric tree graph. For such a graph $\{\gamma_1, \gamma_2, \dots, \gamma_m\} = \partial\Omega \subset V$ boundary vertices, that is, if $d(\nu)$ is the vertex index denotes the number of edges included in this vertex, then $\partial\Omega = \{\nu \in V | d(\nu) = 1\}$.

We assume that no vertex has index 2, or we can consider an equivalent graph with two coincident edges. Therefore, $V \backslash \partial\Omega = \{\nu \in V | d(\nu) > 2\}$.

The problem of identifying the source is reduced to the study of the parabolic equation:

$$u_t - u_{xx} + q(x)u = p(t)h(x) \text{ on } E \times (0, T) \tag{1}$$

$$\begin{cases} \sum_{e_j \sim \nu} \partial u_j(\nu, t) = 0 \text{ at each vertex } \quad \nu \in V \backslash \partial\Omega, \quad t \in [0, T] \\ u(\cdot, t) \text{ are continuous at each vertex for all } \quad t \in [0, T] \end{cases} \tag{2}$$

$$\partial u = f \text{ on } \partial\Omega \times [0, T],\ u|_{t=0} = 0 \text{ on } \Omega \tag{3}$$

The matching conditions are expressed in (2), where $\partial u_j(\nu, \cdot)$ means the derivative of the function u towards the vertex ν, taken along the edge e in the direction from the vertex. In this case, $e_j \sim \nu$ means the edge e_j, entering the vertex ν, and the sum is taken over all edges entering ν. From the point of view of our practical model, this is the law of conservation of the current state.

Let $\mathcal{H} = L^2(\Omega)$ and $\mathcal{F}^T = L^2([0, T]; \mathbb{R}^m)$.

For the direct problem, the result is known [2]:

Theorem 1. *If $f, p \in \mathcal{F}^T$, $q, h \in \mathcal{H}$ then for each $t \in [0, T]$, $u = u^f(\cdot, t) \in \mathcal{H}$ and $u^f \in C([0, T]; \mathcal{H})$, where u^f- is a generalized solution (1)-(3).*

Here $p \in H^1(0, T)$. Application of the boundary control method is reduced to specifying the response operator $\tilde{R}^T : \mathcal{F}^T \to \mathcal{F}^T$ as

$$\left(\tilde{R}^T f\right)(t) = u^f(\cdot, t), \quad t \in [0, T] \tag{4}$$

The inverse problem is to restore the source – the vector $h(\cdot)$, sought by $\tilde{R}^T f$ for all $f \in \mathcal{F}^T$. This also means that we know $\tilde{R}^T f$ for $f \equiv 0$.

Solution (1) - (3) can be written in the form $u = y + z$, taking into account that y and z are solutions to the problems:

$$\begin{cases} y_t - y_{xx} + q(x)y = 0 \text{ on } E \times (0, T) \\ \partial y = f \text{ on } \partial\Omega \times [0, T] \end{cases} \tag{5}$$

$$\begin{cases} z_t - z_{xx} + q(x)z = p(t)h(x) \text{ on } E \times (0, T) \\ \partial z = f \text{ on } \partial\Omega \times [0, T] \end{cases} \tag{6}$$

with y and z, satisfying the Kirchhoff-Neumann matching conditions on $V \backslash \partial\Omega$ and zero initial conditions. Let us represent the solution of equation (5) in the form y^f. Hence, for (5) the response operator $\tilde{R}^T : \mathcal{F}^T \to \mathcal{F}^T$ is given by the formula

$$\left(R^T f\right)(t) = y^f_{|\partial\Omega} = u^f_{|\partial\Omega} = y^0_{|\partial\Omega} = \tilde{R}^T f - \tilde{R}^T 0. \tag{7}$$

The first inverse problem of recovering the vector $q(\cdot)$ from $R^T f$ for all $f \in \mathcal{F}^T$ was solved in [3].

Theorem 2. *The operator* $R^T : \mathcal{F}^T \to \mathcal{F}^T$ *, known for any T>0, uniquely determines the topology, edge lengths and edge potentials* $q_j, \quad j = 1, 2, \ldots, N,$ *of a graph.*

Our main goal is to solve inverse problem (6), with Kirchhoff-Neumann matching conditions on $V \backslash \partial\Omega$, and zero initial conditions. On the basis of *Theorem 2*, the potential q is assumed to be known. We denote the solution z as u^0. Thus, based on our observations

$$\chi(t) = z_{|\partial\Omega} = u^0{}_{|\partial\Omega}$$

and the well-known solution for reconstructing the potentials of the edges $q(\cdot)$ of the graph, our inverse problem is to use observations $\chi(t), t \in [0, T]$ to find the vector $h(\cdot)$ on E.

Let the operator $\mathcal{L}$, be given by the expression

$$\left(\mathcal{L}\phi\right)(x) := -\frac{d^2\phi}{dx^2}(x) + q\left(x\right)\phi\left(x\right) \tag{8}$$

defined on E with domain $D\left(\mathcal{L}\right) = \mathcal{H}^2$. Here $\mathcal{H}^2$ is the space of continuous functions ν on Ω such that $\nu_{|e} \in H^2(e)$ for each $e \in E$, satisfying the Kirchhoff-Neumann matching conditions at each interior vertex, and the boundary condition $\partial\phi_{|\partial\Omega} = 0$. The spectrum of $\mathcal{L}$ s strictly discrete, the eigenvalues $\{\lambda_n\}$ are of finite multiplicity, and the corresponding eigenfunctions $\{\phi_n\}$ form an orthonormal basis in $\mathcal{H}$. It is known that the eigenfunctions are bounded and the estimate $|\lambda_n| + 1 \asymp n^2$ is valid for the eigenvalues.

Under zero boundary conditions for the heat flux, there is no nontrivial solution to the eigenvalue problem associated with (8) for $(\lambda \notin \mathbb{R})$. Therefore, let $\phi^f(x, \lambda)$ be the only solution to the initial-boundary value problem $\mathcal{L}\phi = \lambda\phi$ on E satisfying the Kirchhoff-Neumann matching conditions on the inner vertices, and the boundary conditions

$$\phi^{'}\left(\gamma_j, \lambda\right) = f^j, j = 1, 2, \ldots, m, \quad f = col\left(f^1, f^2, \ldots, f^m\right) \tag{9}$$

The Titchmarsh-Weyl matrix function $(TW), M(\lambda)$, is uniquely determined by the relation

$$\phi^f|_{\partial\Omega} = M(\lambda) f\left(\gamma_j, \lambda\right) = f^j, \quad j = 1, 2, \ldots, m, \quad f = col\left(f^1, f^2, \ldots, f^m\right) \tag{10}$$

The TW-function $M\left(\lambda\right) = \{M_{ij}\}_{i,j=1}^m$, known for $\Im\lambda > 0$, is constructed from our data and is used to solve the inverse problem on the graph.

Using the Kirchhoff-Neumann matching condition and integrating by parts, we give the solution of the initial-boundary value problem with boundary condition (9) in the form

$$\phi^f\left(x, \lambda\right) = \sum_{n=1}^{\infty} \frac{\langle f, \phi_n|_{\partial\Omega}\rangle}{\lambda_n - \lambda} \phi_n\left(x\right) \tag{11}$$

Here $\langle f, \phi_n|_{\partial\Omega}\rangle$ stands for the scalar product in $\mathbb{R}^m$. Therefore, the TW-matrix function $\{M_{ij}\}_{i,j=1}^m$ is defined as follows

$$M\left(\lambda\right) f = \sum_{n=1}^{\infty} \frac{\langle f, \phi_n|_{\partial\Omega}\rangle}{\lambda_n - \lambda} \phi_n|_{\partial\Omega}\ , \quad \text{i. e.} \quad M_{ij}\left(\lambda\right) = \sum_{n=1}^{\infty} \frac{\phi_n\left(\gamma_i\right)\phi_n\left(\gamma_j\right)}{\lambda_n - \lambda} \tag{12}$$

All series in these expressions converge due to the boundedness of the eigenfunctions and the above growth of the eigenvalues.

For the complete construction, we now have to reconstruct the spectral data $(SD) = \{\lambda_n, \phi_n|_{\partial\Omega}\}_{n\in\mathbb{N}}$ from dynamic inverse data (operator R^T), using the coupling operator C^T and spectral controllability of the system (5).

Theorem 3. *For any* $T > 0$ *and for each* $n \in \mathbb{N}$, *there is a control* $f_n \in \mathcal{F}_0^T := H_0^1\left(0, T; \mathbb{R}^m\right)$ *such that* $\nu^{f_n}(\cdot, T) = \phi_n$ *in* Ω. *Controllability can be achieved without using control at any one boundary vertex, that is, we can put, say,* $f^m(t) = 0, \quad t \in [0, T]$.

Control data $f, g \in \mathcal{F}^T$, with y^f and y^g, will be the corresponding solutions (7), and without loss of generality, we will assume that $f^m = g^m = 0$. The communication operator $C^T : \mathcal{F}^T \to \mathcal{F}^T$ in its bilinear form

$$\left(C^T f, g\right)_{\mathcal{F}^T} := \Big(y^f\left(\cdot\ ,\ T\right),\ y^g\left(\cdot\ , T\right)\Big)_{\mathcal{H}}. \tag{13}$$

With the help of (13) we can find the spectral data. Now suppose that in problem (6) we know $q(\cdot)$ on E and we want to restore $h(x)$ on E. Recall that $\chi(t) = z_{|\partial\Omega}$ are our observations, $p \in H^1(0,T)$, $p(0) \neq 0$, and $\mathcal{H} = L^2(\Omega)$. Then the solution to problem (8):

$$z(x,t) = \sum_{n=1}^{\infty} h_n \phi_n(x) \int_0^t p(t-\tau) e^{-\lambda_n t} d\tau \tag{14}$$

where $h_n = (h,\ \phi_n)_{\mathcal{H}}$. Hence,

$$\chi(t) := \int_0^t p(t-s) W(s)\, ds, \tag{15}$$

where

$$W(t) = \sum_{n=1}^{\infty} h_n \phi_n(0) e^{-\lambda_n t} \tag{16}$$

Differentiating (15), we arrive at the Volterra integral equation of the second kind with respect to $W(\cdot)$:

$$\chi'(t) = p(0) W(t) + \int_0^t p'(t-s) W(s)\, ds \tag{17}$$

Theorem 4. *The family* $\left\{\phi_n(0) e^{-\lambda_n t}\right\}_{t\in[0,T]}$ *is minimal on* $\mathcal{F}^T = L^2([0,T];\mathbb{R}^m)$, *for all* $T > 0$ *with biorthogonal sequence* $\{\Theta_n\}$. *Hence,*

$$h_n = (W, \Theta_n)_{\mathcal{F}^T} \qquad h(x) = \sum_{n=1}^{\infty} h_n \phi_n(x).$$

The development of a numerical method for determining Θ_n is quite difficult even in the one-dimensional case. Nevertheless, for a single edge, say e_i, which we identify with the interval $(0,\ l_i)$, we propose a direct approach to finding h_n, and hence $h(x)$. For $h_n = \left(h_{|e_i}, \phi_n\right)_{L^2(e_i)}$ the solution to problem (8) on e_i takes the form

$$z(x,t) = \int_0^t p(\tau) \left(\sum_{n=1}^{\infty} h_n \phi_n(x) e^{-\lambda_n (t-\tau)} \right) d\tau \tag{18}$$

Thence we obtain

$$\chi(t) := z(0,t) = \int_0^t p(s) \left(\sum_{n=1}^{\infty} h_n \phi_n(0) e^{-\lambda_n (t-s)} \right) ds \tag{19}$$

If we put $Z(x,t) := \sum_{n=1}^{\infty} h_n \phi_n(x) e^{-\lambda_n t}$, then Z satisfies the following equation

$$\begin{cases} Z_t - Z_{xx} + q(x) Z = 0 & 0 < x < l, \quad 0 < t < T \\ Z_x(0,t) = 0 = Z_x(l,t) & 0 < t < T \\ Z(x,0) = \sum_{n\geq 1} h_n \phi_n(x) = h(x) & 0 < x < l_i. \end{cases} \tag{20}$$

Thus, $z(x,t) = \int_0^t p(\tau) Z(x, t-\tau) d\tau$, so if we put $r(t) := Z(0,t)$, then

$$\chi(t) = \int_0^t p(t-\tau) r(\tau)\, d\tau \tag{21}$$

Differentiating the expression in (44), we arrive at the Volterra integral equation of the second kind with respect to $r(\cdot)$:

$$\chi'(t) = p(0) r(t) + \int_0^t p'(t-\tau) r(\tau) d\tau \tag{22}$$

Thus, there is a unique solution $r(t)$ for $0 < t < T$. Having $r(t) = \sum_{n=1}^{\infty} r_n e^{-\lambda_n t}$, where $r_n = h_n \phi_n(0)$, and knowing the spectral data $\{\lambda_n, \phi_n(0)\}$, defined values $r_n's$ and determined by the values $h_n's$, and hence the function $h(x)$ on e_i.

Now suppose we got $r(t)$ from solution (22). For a solution for any number $r_n^{'}s$, say, $N \in \mathbb{N}$, we put: $r_N(t) = \sum_{n=1}^{N} r_n e^{-\lambda_n t}$. One way to calculate:

$$V\vec{r} = \begin{bmatrix} 1 & 1 & 1 & \dots & 1 \\ & \lambda_1\lambda_2\lambda_3 & \dots & \lambda_N & \\ & \lambda_1^2\lambda_2^2\lambda_3^2 & \dots & \lambda_N^2 & \\ & & \dots & & \\ & \lambda_1^{N-1}\lambda_2^{N-1}\lambda_3^{N-1} & \dots & \lambda_N^{N-1} & \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \\ r_3 \\ \dots \\ r_N \end{bmatrix} = \begin{bmatrix} r_N(0) \\ -r_N^{'}(0) \\ r_N^{(2)}(0) \\ \dots \\ (-1)^{N-1} r_{N-1}^{(N-1)}(0) \end{bmatrix}.$$

Since V is the Vandermonde determinant and the eigenvalues are prime, V is non-degenerate, so the vector $r_n^{'}s$ can be uniquely found. Another calculation option is to select small t_1 from the interval $(0,T)$, and the assumption that $t_j = jt_1, \quad j = 1, \dots, N$. Then one can define an $N \times N$ matrix $M_N = \left(e^{-\lambda_j t_i}\right)$ and solve the matrix equation $M_{N\vec{r}} = \vec{d}$, where $\vec{d}^T = (r(t_1), r(t_2), \dots, r(t_N))$.

The matrix M_N is non-degenerate, as can be shown, it will be degenerate if and only if some of the $\lambda_n^{'}s$ are equal. In one of two cases, we have $q(x)$ and $h(x)$.

The problem will theoretically be completed when we restore k original g_j parameters of conductivity.

In the initial scaling of the cable equation, we chose arbitrary $i, \quad 1 \le i \le k$ and defined $u = v - E_i$. So the N-vectors u and h defined on $E\times(0,T)$, must be indexed by i; say $u = u^{[i]}$ and $h = h^{[i]}$, where each component $h^{[i]}$ is given as $h_l^{[i]} = \sum_{1 \le j \le k} g_{lj} E_{ji}$. Then, to solve *Inverse Problem 1*, you must first obtain $q(\cdot)$, and then solve the *Inverse Problem 2* k-fold, to find $h^{[i]}, \ i = 1, 2, \dots, k$. The matrix of conductivity parameters takes the form $\mathcal{G} = (g_{jl}) \in \mathbb{R}^{N\times k}$. We define $\mathcal{E} = (e_{ij}) \in \mathbb{R}^{k\times k+1}$, and $e_{i1} = \hat{R}$ and $e_{ij} = E_{i\ j-1}$ for $2 \le j \le k+1, \quad 1 \le i \le k$. If $\mathcal{K} \in \mathbb{R}^{k+1\times k}$ is such that $\mathcal{E}\mathcal{K} = I_k$, where I_k is a $k \times k$ identification matrix, then $\mathcal{G} = [h]\mathcal{K}$, where $[h]$ is an $N \times k$ matrix with the i-th column $h^{[i]}$.

The presented mathematical model is a theoretical substantiation of an inverse problem with a finite number of distributed parameters for a partial differential equation on a tree graph.

The leaf peeling method [4] allows solving the problem sequentially on a segment, on a tree-graph, on a sheaf-graph, over the entire tree-graph.

3. Conclusion

This paper continues the research of famous scientists in the field of control theory and inverse problems on graphs. The most constructive procedures for solving a whole class of inverse problems are reflected in the scientific results of SA Avdonin, his co-authors and his scientific school. They have created a mathematically rigorous approach to control problems and inverse problems for partial differential equations on graphs. A boundary control method (BCM) has been developed based on the relationship between inverse problems (identification) and controllability of dynamic systems: if the system is controllable, then it is identifiable. This method has been successfully applied to almost all linear equations of mathematical physics: the wave equation; heat conduction equations, Maxwell, Schrödinger. BCM advantages: it remains linear at all stages; applicable to a wide range of linear systems; it is essentially independent of the dimension of the system and, finally, allows one to construct simple algorithms and provide stable numerical implementations. A characteristic feature of BCM is its locality. For inverse problems on graphs, this means that only data related to this subgraph is required to restore the topology and other parameters of a subgraph. This property provides the advantage of BCM over other methods and allows us to extend the proposed approach from interval to graphs when solving inverse problems of mathematical physics. Another distinctive feature of BCM lies in a variety of interdisciplinary connections: in addition to partial differential equations, controllability theory of systems, asymptotic methods, complex analysis, functional analysis, operator theory, Banach algebras, etc. are used.

Another important method is the leaf peeling method, developed by S. Avdonin and P. Kurasov in [2]. This method assumes a sequential procedure for restoring tree parameters from vertices to the root.

In [5], the leaf peeling method was successfully applied to inverse boundary value problems with non-standard equations at the vertices, and in [6-9] - to two-speed wave equations on tree-type graphs. The leaf peeling method allows one to reduce the control problem and the inverse problem to the integral equations of Volterra or Fredholm with subsequent numerical implementation.

Spectral and dynamic inverse problems for parabolic, wave and Schrödinger equations were solved [4] on graphs without cycles.

Source identification problems for the wave equation on graphs were solved in [10].

In [11, 3], inverse problems were considered for parabolic equations of the form: $u_t - u_x x + q(x)u = p(t)h(x)$. At the inner vertices of the tree graph, Kirchhoff-Neumann matching conditions are specified. From the point of view of the neural model, this is the law of conservation of currents. The problem of recovering the topology of the graph, the lengths of the edges, as well as the potential q and the source h on the edges of the graph is solved from the dynamic inverse data.

In this paper, we continued to develop the ideas of these [2]-[11] papers from the point of view of practical application in the diagnosis of congestion in oil pipelines.

References

1 Правила эксплуатации магистральных нефтепроводов, утвержденные приказом Министра энергетики Республики Казахстан от 29 октября 2014 года № 84 [Электронный ресурс]. -URL: https://adilet.zan.kz/rus/docs/V1400010107 (Дата обращения: 01.06.2021).

2 Avdonin S., Kurasov P. Inverse problems for quantum trees // Inverse Problem and Imaging, 2008. - № 2 (1). – P. 1-21.

3 Avdonin S., Bell J., Nurtazina K. Determining distributed parameters in a neuronal cable model on a tree graph // Mathematical methods in applied sciences, 2017. - № 40 (11). - P. 3973-3981.

4 Avdonin S.A., Mikhaylov V.S., Nurtazina K.B. On Inverse Dynamical and Spectral Problems for the. Wave and Schrodinger Equations on Finite Trees. The Leaf Peeling Method // Journal of Mathematical Sciences (USA), 2017. Vol. 224, No. 1. – P.1-10.

5 Avdonin S., Kurasov P. and Nowaczy M. On the reconstruction of boundary conditions for star graphs // Inverse Problems and Imaging, 2010. Vol.4, No. 4. – P. 579-598.

6 S. Avdonin, G. Leugering and V. Mikhaylov, On an inverse problem for tree-like networks of elastic strings, ZAMM Z. Angew. Math. Mech., 90 (2010), no 2, 136-150.

7 Avdonin S., Abdon Ch.R., Leugering G., Mikhaylov V. On the inverse problem of the two-velocity tree-like graph // ZAMM Z.Angew. Math. Mech. 1–11 (2015).

8 Avdonin S.A.,Blagoveshchensky A.S.,Choque-Rivero A.E.,Mikhaylov V.S. Dynamical inverse problem for two-velocity systems on finite trees // Proceedings of the International Conference Days on Diffraction, DD 2016. – P. 25-30.

9 S. Avdonin, A. Choque Rivero, G. Leugering and V. Mikhaylov, On the inverse problem of the two velocity tree-like graph, Zeit. Angew. Math. Mech., (2015), 95, no. 12, 1490-1500.

10 Avdonin S., Nicaise S. Source identification problems for the wave equation on graphs // Inverse Problems, 2015. Vol. 31. 095007 (29 pp).

11 Avdonin S., Bell J. Determining a Distributed Condactance Parameter for a Neuronal Cable Model on a Tree Graph // Inverse Problems and Imaging, 2015. Vol. 9, No. 3. – P. 645-659.

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Мұнай құбырының техникалық жағдайын диагностикалаудың математикалық негіздемесі

Аннотация. Мақалада граф-ағаштағы параболалық типтегі дербес туындылардағы дифференциалдық теңдеу үшін көзді сәйкестіндірудің математикалық моделі ұсынылған. Әзірленген модель сұйықтықтарды айдау жүзеге асырылатын ауқымды құбыржолдардың пайдаланылуын диагностикалауда, атап айтқанда, магистральдық мұнай құбырының жеке буынында кептелісті анықтауда нәтижелерді қолдануға мүмкіндік береді.

Түйін сөздер: магистральдық мұнай құбыры, мұнай құбыры буынындағы кептеліс, жылу теңдеуі, граф-ағаш, көзді сәйкестендіру, шекаралық басқару әдісі, leaf peeling әдісі.

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Математическое обоснование диагностики технического состояния нефтепровода

Аннотация. В статье предлагается математическая модель восстановления источника для дифференциального уравнения в частных производных параболического типа на графе-дереве. Разработанная модель позволяет применить результаты в диагностике эксплуатации масштабных трубопроводов, по которым осуществляется перекачка жидкостей, в частности, обнаружения затора в отдельном звене магистрального нефтепровода.

Ключевые слова: магистральный нефтепровод, затор в звене нефтепровода, тепловое уравнение, граф-дерево, идентификация источника, метод граничного управления, leaf peeling method.

References

1 Pravila ekspluatacii magistral'nyh nefteprovodov, utverzhdennye prikazom Ministra energetiki Respubliki Kazahstan ot 29 oktyabrya 2014 goda No 84 [Rules for the operation of trunk oil pipelines approved by order of the Minister of Energy of the Republic of Kazakhstan dated October 29, 2014 No. 84] [Electr. resourse]. Available at: https://adilet.zan.kz/rus/docs/V1400010107 (Accessed: 01.06.2021).
2 Avdonin S., Kurasov P. Inverse problems for quantum trees, Inverse Problem and Imaging, 1(2), 1-21(2008).
3 Avdonin S., Bell J., Nurtazina K. Determining distributed parameters in a neuronal cable model on a tree graph, Mathematical methods in applied sciences, 11(40), 3973-3981(2017).
4 Avdonin S.A., Mikhaylov V.S., Nurtazina K.B. On Inverse Dynamical and Spectral Problems for the. Wave and Schrodinger Equations on Finite Trees. The Leaf Peeling Method, Journal of Mathematical Sciences (USA), 224(1), 1-10(2017).
5 Avdonin S., Kurasov P. and Nowaczy M. On the reconstruction of boundary conditions for star graphs, Inverse Problems and Imaging, 4(4), 579-598(2010).
6 Avdonin S., Leugering G. and Mikhaylov V. On an inverse problem for tree-like networks of elastic strings, ZAMM Z. Angew. Math. Mech., 90(2), 136-150(2010).
7 Avdonin S., Abdon Ch.R., Leugering G., Mikhaylov V. On the inverse problem of the two-velocity tree-like graph, ZAMM Z.Angew. Math. Mech. 1–11 (2015).
8 Avdonin S.A.,Blagoveshchensky A.S.,Choque-Rivero A.E.,Mikhaylov V.S. Dynamical inverse problem for two-velocity systems on finite trees, Proceedings of the International Conference Days on Diffraction, DD 2016. P. 25-30.
9 Avdonin S., Choque Rivero A., Leugering G. and Mikhaylov V. On the inverse problem of the two velocity tree-like graph, Zeit. Angew. Math. Mech., 95(12), 1490-1500(2015).
10 Avdonin S., Nicaise S. Source identification problems for the wave equation on graphs, Inverse Problems, 2015. Vol. 31. 095007 (29 pp).
11 Avdonin S., Bell J. Determining a Distributed Condactance Parameter for a Neuronal Cable Model on a Tree Graph, Inverse Problems and Imaging, 9(3), 645-659(2015).

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