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Γ-supercyclicity for Bilateral Shift Operators and Translation Semigroups

Abstract: We characterize the Γ-supercyclicity of bilateral shift operators and translation semigroups in weighted L^p spaces in terms of the weight and Γ. We survey the notion of S-density introduced by E. Abakumov and Y. Kuznetsova. Finally, we present without proof some characterization of a supercyclic version for the S-density of translation operator.

Keywords: backward shift, S-density, Hypercyclicity, Supercyclicity, Γ-supercyclicity, Translation semigroup.

1. Introduction

In this note we will deal with hypercyclicity and supercyclicity of an important class of operators in dynamics of linear operators. More precisely, we discuss some characterization of hypercyclicity and supercyclicity of translation operator in weighted L^p space. Let us assume that X is an infinite-dimensional Banach space, let $\mathcal{L}(X)$ be the space of bounded linear operators on X, and let Γ be a subset of the complex plane such that $\Gamma \setminus \{0\}$ is non-empty. An operator $T \in \mathcal{L}(X)$ is said to be **hypercyclic** if there exists some vector $x \in X$ whose orbit under T, i.e., $\mathrm{Orb}(x, T) := \{T^n x : n \in \mathbb{N}\}$ is dense in X. Similarly, T is called **supercyclic** if the projective orbit of some vector $x \in X$ under T, i.e., $\mathrm{Orb}(\mathbb{C}x, T) := \{\lambda T^n x : n \in \mathbb{N}, \lambda \in \mathbb{C}\}$ is dense in X, more generally, T is said to be Γ-supercyclic if there exists a vector $x \in X$ such that the orbit of Γx under T, i.e., $\mathrm{Orb}(\Gamma x, T) := \{\lambda T^n x : n \in \mathbb{N}, \lambda \in \Gamma\}$ is dense in X. The vector x is called a Γ-supercyclic vector for T and the set $\Gamma\text{-}\mathcal{SC}(T)$ stands for the set of all Γ-supercyclic vectors for T. It easy to check that the set of Γ-supercyclic vectors for T is either empty or dense in X. Indeed, if x is a Γ-supercyclic vector for T, then the orbit of Γx is contained in $\Gamma\text{-}\mathcal{SC}(T)$. The notion of Γ-supercyclicity was introduced recently in [4]. For more information about hypercyclicity and supercyclicity, see [3,10]. Hypercyclicity and supercyclicity of strongly continuous semigroups have also been considered. Recall that a strongly continuous semigroup of operators - or C_0-semigroup - on X is a family $(T_t)_{t\geqslant 0}$ of bounded linear operators on X that satisfies $T_0 = I$, $T_{t+s} = T_t T_s$ for all $t, s \geqslant 0$, and such that the map $\mathbb{R}_+ \longrightarrow X;\ t \longmapsto T_t x$ is continuous for every $x \in X$. Hypercyclicity and supercyclicity of C_0-semigroups are defined in a way similar to the discrete case. A C_0-semigroup $\mathcal{T} = (T_t)_{t\geqslant 0}$ on X is hypercyclic if there exists a vector $x \in X$ whose orbit under $\mathcal{T}$, $\mathrm{Orb}(x, \mathcal{T}) := \{T_t x : t \geqslant 0\}$, is dense in X, and $\mathcal{T}$ is supercyclic if there exists $x \in X$ such that the set $\mathrm{Orb}(\mathbb{C}x, \mathcal{T}) := \{\lambda T_t x : t \geqslant 0, \lambda \in \mathbb{C}\}$ is dense in X. Finally Γ-supercyclicity for strongly continuous semigroups is defined in a natural way as in [2]. Characterizations for hypercyclicity and supercyclicity for weighted shift operators were obtained by Salas in [12,13], while hypercyclicity of translation semigroups was characterized by Desch, Schappacher and Webb [8]. Moreover, Matsui, Yamada, and Takeo gave a characterization of supercyclicity of translation semigroups [11]. In this paper, we will characterize the Γ-supercyclicity of bilateral shift operators (see Theorem A, see Section 2) and of translation semigroups (see Theorem B, see Section 3). Next, after a short survey on some of the results of Abakumov and Kuznetsova in [1] (Section 4), we state several results obtained jointly with Y. Kuznetsova which generalize some of those contained, e.g., in [6,9,11,13] (Section 5). These results will be fully developped and proved in a forthcoming paper.

2. Γ-SUPERCYCLICITY OF BILATERAL SHIFT OPERATORS

As mentioned above, characterizations of hypercyclicity and supercyclicity of weighted shift operators were obtained by Salas in [12, 13]. In this section, we suppose that $\omega = (\omega_n)_{n\in\mathbb{Z}}$ is a sequence of positive numbers such that $\sup\limits_{n\in\mathbb{Z}} \frac{\omega_n}{\omega_{n+1}} < +\infty$, $1 \leqslant p < +\infty$, and B is the unweighted backward shift acting on $\ell^p(\mathbb{Z},\omega)$ defined by

$$B: \begin{array}{ccc} \ell^p(\mathbb{Z},\omega) & \longrightarrow & \ell^p(\mathbb{Z},\omega) \\ (x_k)_{k\in\mathbb{Z}} & \longmapsto & (x_{k+1})_{k\in\mathbb{Z}} \end{array}, \tag{1}$$

where

$$\ell^p(\mathbb{Z},\omega) := \{x = (x_k)_{k\in\mathbb{Z}} \in \mathbb{C}^{\mathbb{Z}} : \|x\|^p_{p,\omega} = \sum_{k\in\mathbb{Z}} |x_k|^p \omega_k^p < +\infty\}.$$

We quote below characterizations of hypercyclicity and supercyclicity of the backward shift obtained in [3, Theorem 1.38].

Theorem 1 (Salas). *Let B be given by* (1)

1. *B is hypercyclic if and only if for any $q \in \mathbb{N}$, $\liminf\limits_{n\to+\infty} \omega_{\pm n+q} = 0$.*
2. *B is supercyclic if and only if for any $q \in \mathbb{N}$, $\liminf\limits_{n\to+\infty} \omega_{n+q}\omega_{-n+q} = 0$.*

Inspired by the proof of Salas´ theorem, we obtain the following more general characterization of Γ-supercyclicity for backward shift operators.

Theorem A. Let $\Gamma \subset \mathbb{C}$ be such that $\Gamma \setminus \{0\}$ is non-empty. Then B is Γ-supercyclic if and only if for any $q \in \mathbb{N}$, there exist $(\lambda_k)_{k\in\mathbb{N}} \subset \Gamma \setminus \{0\}$ and $(n_k)_{k\in\mathbb{N}} \subset \mathbb{N}$,with $n_k \to \infty$ as $k \to \infty$, such that

$$\lim_{k\to+\infty} \max\left\{\frac{1}{|\lambda_k|}\omega_{n_k+q}\,,\,|\lambda_k|\omega_{-n_k+q}\right\} = 0.$$

Proof. $(\Rightarrow)$: Fix $q \in \mathbb{N}$ and $1 < \delta < min\{1,\omega_q\}$. Since the set $\Gamma\text{-}\mathcal{SC}(B)$ of Γ-supercyclic vectors of B is dense in $\ell^p(\mathbb{Z},\omega)$, there exist $x \in \Gamma\text{-}\mathcal{SC}(B)$, $\lambda \in \Gamma\setminus\{0\}$ and an integer $n > 2q$ such that:

$$\|x - e_q\|_{p,\omega} < \delta \quad \text{ and } \|\lambda B^n x - e_q\|_{p,\omega} < \delta,$$

where $e_q = (\delta_{q,k})_{k\in\mathbb{Z}}$ with $\delta_{q,k}$ the Kronecker delta. Regarding the q-th and $(n+q)$-th coordinate in the first inequality, we obtain:

$$|x_q - 1|\omega_q < \delta \quad \text{ and } \quad |x_{n+q}|\omega_{n+q} < \delta.$$

Similarly, regarding the q-th and $(-n+q)$-th coordinate in the second inequality, we obtain:

$$|\lambda x_{n+q} - 1|\omega_q < \delta \quad \text{ and } \quad |\lambda||x_q|\omega_{-n+q} < \delta.$$

Combining these inequalities, we get

$$\max\left\{\frac{1}{|\lambda|}\omega_{n+q}\,,\,|\lambda|\omega_{-n+q}\right\} < \frac{\delta\omega_q}{\omega_q - \delta}.$$

Since δ is arbitrary, this establishes the necessary condition.

$(\Leftarrow)$: Fix $C > \max\{1, \sup \frac{\omega_n}{\omega_{n+1}}\}$. We can find two sequences $(n_k)_{k\in\mathbb{N}} \subset \mathbb{N}$ and $(\lambda_k)_{k\in\mathbb{N}} \subset \Gamma \setminus \{0\}$ such that, for any $l \in \mathbb{Z}$

$$\lim_{k\to+\infty} \max\left\{\frac{1}{|\lambda_k|}\omega_{n_k+l}\,,\,|\lambda_k|\omega_{-n_k+l}\right\} = 0. \tag{2}$$

Indeed, it is clear that there exist two sequences $(n_k)_{k\in\mathbb{N}} \subset \mathbb{N}$ and $(\lambda_k)_{k\in\mathbb{N}} \subset \Gamma\setminus\{0\}$ such that $\max\left\{\frac{1}{|\lambda_k|}\omega_{n_k+k}\,,\,|\lambda_k|\omega_{-n_k+k}\right\} < C^{-3k}$. Fix $l \in \mathbb{Z}$ and let $k \in \mathbb{N}$ such that $k > |l|$. Since

$$\omega_{\pm n_k+l} = \frac{\omega_{\pm n_k+l}}{\omega_{\pm n_k+l+1}} \cdots \frac{\omega_{\pm n_k+k-1}}{\omega_{\pm n_k+k}}\omega_{\pm n_k+k},$$

we obtain $\max\left\{\frac{1}{|\lambda_k|}\omega_{n_k+l}\,,\,|\lambda_k|\omega_{-n_k+l}\right\} < C^{-k} \underset{k\to+\infty}{\longrightarrow} 0$.

Let $Y := \{y_j : j \geqslant 1\} \subset c_{00}(\mathbb{Z})$ (the linear span of the basis vectors e_i) be a dense subset of $\ell^p(\mathbb{Z},\omega)$. We can assume that for every $j \geqslant 1$, $y_j := \sum_{|m|\leqslant q_j} y_j(m)e_m$ where $q_j \in \mathbb{N}$ and $y_j(m) \in \mathbb{C}$. Let also $S : Y \longrightarrow \ell^p(\mathbb{Z},\omega)$ be the map defined by $S(x_n)_{n\in\mathbb{Z}} = (x_{n-1})_{n\in\mathbb{Z}}$ (i.e., $Se_n = e_{n+1}$). We will construct by induction a sequence $(k_j)_{j\geqslant 1} \subset \mathbb{N}$ such that for every $j \geqslant 1$:

(a) $\dfrac{1}{|\lambda_{k_j}|}\|S^{n_{k_j}} y_j\|_{p,\omega} < 2^{-j}$.

(b) For every $l = 1, ..., j-1$, $\dfrac{|\lambda_{k_j}|}{|\lambda_{k_l}|}\|B^{n_{k_j}} S^{n_{k_l}} y_l\|_{p,\omega} < 2^{-j}$.

(c) For every $l = 1, ..., j-1$, $\dfrac{|\lambda_{k_l}|}{|\lambda_{k_j}|}\|B^{n_{k_l}} S^{n_{k_j}} y_j\|_{p,\omega} < 2^{-j}$.

Assume that $k_1, ..., k_{j-1}$ have been chosen. For all $n \in \mathbb{N}$ and $l \in \{1, ..., j-1\}$, an easy computation gives that $\|S^n y_j\|^p_{p,\omega} \leqslant \omega^p_{n+q_j} C_j$, where C_j is a constant (depending only on y_j, C and p) and

$$\|B^n S^{n_{k_l}} y_l\|^p_{p,\omega} \leqslant \omega^p_{-n+n_{k_l}+q_l} C_l \quad \text{and} \quad \|B^{n_{k_l}} S^n y_j\|^p \leqslant \omega^p_{n-n_{k_l}+q_j} C_j.$$

Combining the last three inequalities with (2), we can then choose k_j sufficiently large in order to satisfy the conditions (a), (b) and (c).

From (a), we get that the series $\sum_{l\geqslant 1} \dfrac{1}{\lambda_{k_l}} S^{n_{k_l}} y_l$ converges. Moreover $x := \sum_{l\geqslant 1} \dfrac{1}{\lambda_{k_l}} S^{n_{k_l}} y_l$ is a Γ-supercyclic vector for B. Indeed, for every $j \geqslant 1$, and by (b) and (c), we have

$$\|\lambda_{k_j} B^{n_{k_j}} x - y_j\|_{p,\omega} \leqslant \sum_{l=1}^{j-1} \frac{|\lambda_{k_j}|}{|\lambda_{k_l}|}\|B^{n_{k_j}} S^{n_{k_l}} y_l\|_{p,\omega} + \sum_{l=j+1}^{+\infty} \frac{|\lambda_{k_j}|}{|\lambda_{k_l}|}\|B^{n_{k_j}} S^{n_{k_l}} y_l\|_{p,\omega}$$

$$\leqslant \frac{j-1}{2^j} + \sum_{l=j+1}^{+\infty} \frac{1}{2^l} = \frac{j+1}{2^j} \underset{j\to+\infty}{\longrightarrow} 0.$$

This finishes the proof.

Another way to prove the sufficient condition of the previous Theorem without constructing a Γ-supercyclic vector is by using the following result. It can be proved by extending the proof of the supercyclicity criterion [3, Theorem 1.14].

Theorem 2 (Γ-supercyclicity criteria)**.** *Let X be an infinite-dimensional Banach space, $T \in \mathcal{L}(X)$ and let $\Gamma \subset \mathbb{C}$ be such that $\Gamma \setminus \{0\}$ is non-empty. Assume that there exist an increasing sequence of integers $(n_k)_{k\in\mathbb{N}}$, a sequence $(\lambda_k)_{k\in\mathbb{N}} \subset \Gamma \setminus \{0\}$, two dense subsets X_0 and Y_0 in X and a family of applications $S_{n_k} : Y_0 \longrightarrow X$ such that, for any $x \in X_0$ and any $y \in Y_0$, the following conditions hold:*

1. $\lambda_k T^{n_k} x \underset{k\to+\infty}{\longrightarrow} 0$;
2. $\dfrac{1}{\lambda_k} S_{n_k} y \underset{k\to+\infty}{\longrightarrow} 0$;
3. $T^{n_k} S_{n_k} y \underset{k\to+\infty}{\longrightarrow} y$;

Then T is Γ-supercyclic.

3. Γ-SUPERCYCLICITY OF TRANSLATION SEMIGROUPS

Hypercylicity and supercyclicity of translation semigroups are characterized respectively in [8, 11]. In this section, we assume that $\omega : \mathbb{R} \longrightarrow \mathbb{R}$ is an **admissible weight**, i.e., a positive measurable function such that there exist $M \geqslant 1$ and $a \in \mathbb{R}$ such that $\omega(x) \leqslant Me^{at}\omega(x+t)$ for all $x \in \mathbb{R}$ and $t > 0$. For $1 \leqslant p < +\infty$, we now consider the weighted Lebesgue space

$$\mathrm{L}^p(\mathbb{R},\omega) := \left\{ f : \mathbb{R} \longrightarrow \mathbb{C} : f \text{ measurable }, \int_{\mathbb{R}} |f(t)|^p \omega(t)^p \, \mathrm{d}t < \infty \right\}$$

endowed with the norm

$$\|f\|_{p,\omega} := \left(\int_{\mathbb{R}} |f(t)|^p \omega(t)^p \, \mathrm{d}t \right)^{1/p} .$$

The translation semigroup $\mathcal{T} = (T_t)_{t \geqslant 0}$ on $\mathrm{L}^p(\mathbb{R}, \omega)$ is defined by:

$$(T_t f)(x) = f(x+t), \quad f \in \mathrm{L}^p(\mathbb{R}, \omega), \, x \in \mathbb{R} \text{ and } t \geqslant 0.$$

Note that the translation semigroup on $L^p(\mathbb{R}, \omega)$ is a strongly continuous semigroup [8, Lemma 4.6]. Hypercyclicity of translation semigroup was characterized in terms of the weight by Desch, Schappacher and Webb:

Theorem 3 ([8, Theorem 4.8]). *Let $\mathcal{T} = (T_t)_{t \geqslant 0}$ be the translation semigroup on $L^p(\mathbb{R}, \omega)$. Then $\mathcal{T}$ is hypercyclic if and only if for each $\theta \in \mathbb{R}$, there exists a sequence $(t_j)_{j \geqslant 0} \subset \mathbb{R}^+$ ($t_j \underset{j \to +\infty}{\longrightarrow} +\infty$) such that*

$$\lim_{j \to +\infty} \max\{\omega(t_j + \theta), \omega(-t_j + \theta)\} = 0.$$

Similarly, the supercyclicity of translation semigroup was characterized by Matsui, Yamada and Takeo.

Theorem 4 ([11, Theorem 1]). *Let $\mathcal{T} = (T_t)_{t \geqslant 0}$ be the translation semigroup on $L^p(\mathbb{R}, \omega)$. Then $\mathcal{T}$ is supercyclic if and only if for each $\theta \in \mathbb{R}$, there exists a sequence $(t_j)_{j \geqslant 0} \subset \mathbb{R}^+$ ($t_j \underset{j \to +\infty}{\longrightarrow} +\infty$) such that*

$$\lim_{j \to +\infty} \omega(t_j + \theta) \, \omega(-t_j + \theta) = 0.$$

Inspired by the proof of [11, Theorem 1] or [8, Theorem 4.8] and by using [2, Proposition 3.6 and 3.7], we obtain the following characterization of Γ-supercyclicity of the translation semigroups, similar to Theorem A.

Theorem B. Let $\mathcal{T} = (T_t)_{t \geqslant 0}$ be the translation semigroup on $L^p(\mathbb{R}, \omega)$ and let $\Gamma \subset \mathbb{C}$ with $\Gamma \setminus \{0\}$ non-empty. Then $\mathcal{T}$ is Γ-supercyclic if and only if for each $\theta \in \mathbb{R}$, there exist two sequences $(t_j)_{j \geqslant 0} \subset \mathbb{R}^+$, $t_j \to \infty$ as $j \to \infty$, and $(\lambda_j)_{j \geqslant 0} \subset \Gamma \setminus \{0\}$ such that

$$\lim_{j \to +\infty} \max \left\{ \frac{1}{|\lambda_j|} \omega(t_j + \theta) \, , \, |\lambda_j| \omega(-t_j + \theta) \right\} = 0$$

We finish this section by asking the following open question.

Question. Can we characterize the subsets Γ of the complex plane such that the following condition holds: For every separable complex Banach space X and for every C_0-semigroup $\mathcal{T} = (T_t)_{t \geqslant 0}$ on X, $\mathcal{T}$ Γ-supercyclic implies that for every $t > 0$, T_t is Γ-supercyclic?

This question has a positive answer when $\Gamma = \{1\}$ (see [7]) and $\Gamma = \mathbb{C}$ (see [14]).

4. S-density of translates operators on $L^p(G, \omega)$

All the results in this section were obtained by E. Abakumov and Y. Kuznetsova in [1]. In the remainder of this note, let us assume that G is a locally compact group with identity e and associated left Haar measure μ. Let $1 \leqslant p < +\infty$ and let $\omega : G \longrightarrow \mathbb{R}_+$ be a **weight** on G, i.e., a positive locally p-integrable function (i.e., $\omega \in L^p(K)$ for all compact subsets K of G). Consider the weighted L^p-space

$$L^p(G, \omega) := \left\{ f : G \longrightarrow \mathbb{C} : \, f \text{ measurable }, \|f\|^p_{p,\omega} := \int_G |f(t)|^p \omega(t)^p \, \mathrm{d}\mu(t) < \infty \right\}.$$

For every $s \in G$, if $\operatorname*{ess\,sup}_{t \in G} \dfrac{\omega(st)}{\omega(t)} < +\infty$, then the left translation operator T_s defined on $L^p(G, \omega)$ into itself by

$$(T_s f)(t) = f(s^{-1}t); \, t \in G, \; f \in L^p(G, \omega)$$

is continuous and

$$\|T_s\| = \operatorname*{ess\,sup}_{t\in G} \frac{\omega(st)}{\omega(t)}.$$

In the following, fix $S \subset G$, and assume that ω is an S-**admissible** weight on G, i.e., $\operatorname*{ess\,sup}_{t\in G} \frac{\omega(st)}{\omega(t)} < +\infty$ for all $s \in S$.

Definition 1 ([1]). A function $f \in L^p(G,\omega)$ is called S**-dense** if its S-orbit

$$Orb_S(f) := \{T_s f : s \in S\}$$

is dense in $L^p(G,\omega)$.

Remark that if S is the subsemigroup generated by $s_0 \in G$, then the S-density is equivalent to the hypercyclicity of the operator T_{s_0}. Before giving the complete characterization of the existence of S-dense vectors in $L^p(G,\omega)$, let us denote $\|\omega\|_{p,K}^p := \int_K \omega^p(t)\mathrm{d}\mu(t)$ for any compact K of G. We note that without any condition on the subset S of G, we have the following S-density criteria.

Theorem 5 ([1, Theorem 5]). *The following conditions are equivalent.*

1. *There is an S-dense vector in $\mathrm{L}^p(G,\omega)$.*
2. *For every increasing sequence of compact subsets $\{F_n\}_{n\geqslant 1}$ of G and for every sequence $(\delta_n)_{n\geqslant 1}$ of positive numbers, there is a sequence $(s_n)_{n\geqslant 1} \subset S$ and a sequence of compact subsets $K_n \subset F_n$ such that $s_n^{-1}F_n$ are pairwise disjoint, $\mu(F_n \setminus K_n) < \delta_n$ and,*

$$\sum_{n,k\geqslant 0; n\neq k} \|\omega\|_{p,s_n s_k^{-1} K_k}^p < +\infty.$$

with $s_0 = e$ and $K_0 = \emptyset$.

If the subgroup generated by S is abelian, then we have the following result:

Theorem 6 ([1, Theorem 10]). *The following conditions are equivalent:*

1. *There is an S-dense vector in $L^p(G,\omega)$ for every (resp. some) p, $1 \leqslant p < +\infty$.*
2. *For every compact set $F \subset G$ and any given $\delta > 0$, there exist $s \in S$ and a compact set $K \subset F$ such that $\mu(F \setminus K) < \delta$ and*

$$\operatorname*{ess\,sup}_{sK\cup s^{-1}K} \omega < \delta.$$

This Theorem generalizes Theorem 1 (1) and Theorem 3. Indeed, if G is an infinite countable discrete group, then condition (2) of the above Theorem becomes the following: For every finite subset F of G and any given $\delta > 0$, there exists $s \in S$ such that

$$\max_{t\in sF\cup s^{-1}F} \omega(t) < \delta.$$

The following result extends the characterization of hypercyclicity for invertible bilateral weighted shifts given by Feldman [9].

Proposition. *Assume that G is abelian and ω is a G-admissible weight. There exists an S-dense vector in $L^p(G,\omega)$ if and only if $\inf_{s\in S} \max\{\omega(s), \omega(s^{-1})\} = 0$.*

5. Γ-supercyclic version of S-density

The results of this section are part of a joint work with Y. Kuznetsova and they will be discussed in details in a forthcoming paper. Here we will present some extension of the supercyclic version of the S-density. Under the hypothesis of the previous section, we will assume that G is not compact and ω is a continuous S-admissible weight. To give an extension of Theorem 5, we will introduce the following definition.

Definition 2. Let $\Gamma \subset \mathbb{C}$ be such that $\Gamma \setminus \{0\}$ is non-empty. We say that $f \in L^p(G,\omega)$ is a (Γ, S)**-dense** vector in $L^p(G,\omega)$ if its (Γ, S)-orbit

$$\mathrm{Orb}_S(\Gamma f) := \{\lambda T_s f : \, s \in S, \, \lambda \in \Gamma\}$$

is dense in $L^p(G,\omega)$.

We clearly have the following chain of implications:

$$S\text{-density} \Rightarrow (\Gamma, S)\text{-density} \Rightarrow (\mathbb{C}, S)\text{-density}.$$

We are now able to present the (Γ, S)-density criteria.

Theorem 7. *The following conditions are equivalent:*

1. *There is a (Γ, S)-dense vector in $\mathrm{L}^p(G,\omega)$.*
2. *For every increasing sequence of compact subsets $\{F_n\}_{n\geqslant 1}$ of G and for every sequence $(\delta_n)_{n\geqslant 1}$ of positive numbers, there are sequences $(s_n)_{n\geqslant 1} \subset S$, $(\lambda_n)_{n\geqslant 1} \subset \Gamma \setminus \{0\}$ and a sequence of compact subsets $K_n \subset F_n$ such that $s_n^{-1}F_n$ are pairwise disjoint, $\mu(F_n \setminus K_n) < \delta_n$ and,*

$$\sum_{n,k\geqslant 0; n\neq k} \frac{|\lambda_n|^p}{|\lambda_k|^p} \|\omega\|^p_{p, s_n s_k^{-1} K_k} < +\infty.$$

with $s_0 = e$, $\lambda_0 = 1$ and $K_0 = \emptyset$.

The following Theorem extends Theorem 1 (2).

Theorem 8. *If the subgroup generated by S is abelian, then the following conditions are equivalent:*

1. *For every (some) $p \geqslant 1$, there is a $(\mathbb{C}, S)$-dense vector in $\mathrm{L}^p(G,\omega)$.*
2. *For any compact subset $F \subset G$ and $\delta > 0$, there are $s \in S$ and a compact subset $K \subset F$ such that $\mu(F \setminus E) < \delta$ and*

$$\sup_{t\in K} \omega(st) \sup_{t\in K} \omega(s^{-1}t) < \delta. \tag{3}$$

The following result extends the characterization of supercyclicity for invertible bilateral weighted shifts obtained by Feldman [9].

Proposition 1. *If G is abelian and ω is a G-admissible weight, then there exists a $(\mathbb{C}, S)$-dense vector in $L^p(G,\omega)$ if and only if $\inf_{s\in S} \omega(s)\omega(s^{-1}) = 0$.*

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Екіжақты ығысулар операторлары мен жартылай топтар трансляциясы үшін Γ-суперциклдылық

Аннотация: Мақалада салмақты L^p кеңістігінде салмақ және Γ терминдері мағынасында екіжақты ығысу операторы мен трансляция жартылай топтарының Γ-суперциклдылығына сипаттау берілді. Э. Абакумов және Ю. Кузнецова енгізген S-тығыздық қарастырылады. Трансляция операторының S-тығыздығы үшін суперциклділік нұсқасының бір сипаттамасы дәлелдеусіз беріледі.

Түйін сөздер: Кері ығысу, S-тығыздық, гиперциклділік, суперциклділік, Γ-суперциклділік, трансляциялар жартылай тобы.

Арафат Аббар

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Γ-суперцикличность для операторов двусторонних сдвигов и полугрупп трансляции

Abstract: В статье в весовых пространствах L^p в терминах веса и Γ проведена характеризация Γ-суперцикличности операторов двустороннего сдвига и полугрупп трансляции. Рассматривается понятие S-плотности, введенное Э. Абакумовым и Ю. Кузнецовой. Приводится без доказательства некоторая характеристика суперциклической версии для S-плотности оператора трансляции.

Keywords: Обратный сдвиг, S-плотность, гиперцикличность, суперцикличность, Γ-суперцикличность, полугруппа трансляций.

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