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Linear Differential Evasion Game with Locally Inertial Controls

Аннотация: Linear differential evasion game with two players, the pursuer and the evader, is considered. New classes of controls for both players are introduced, which called locally inertial controls. In these classes the evasion problem from any initial position that does not belong to the terminal set is solved. Effective sufficient conditions of evasion and algorithms for building the evasion control are obtained. The results demonstrated on the well-known in the theory of differential games example.

Ключевые слова: evasion, pursuer, evader, control functions, locally inertial control, terminal set.

1. Introduction. Statement of the evasion problem

Let the motion of an object z is described by linear system of differential equations

$$\dot{z} = Cz - u + v + a, \tag{1}$$

where $z \in R^n$, control parameters $u \in P \subset R^n$, $v \in Q \subset R^n$, C is an $n \times n$ constant matrix, P and Q are nonempty compacts, $a \in R^n$ is a fixed vector.

Terminal set M is a subspace of R^n , $\dot{z}$ is the derivative with respect to time. The object z is controlled by two sides, traditionally called players. The first player (pursuer) acts on the system (1) by selecting some function $u = u(t) \in P$, $t \geq 0$ from the specified allowed class of functions U . The second player (evader) chooses his controls in the form of some function $v = v(t) \in Q$ from his allowed class of functions V .

The following evasion problem is considered: for each initial position $z_0 \notin M$ it is necessary to specify (find) the algorithm of such a choice of control $v^*(t) \in Q, t \geq 0$ and $v^*(\cdot) \in V$, which provides for the evader avoiding of trajectory $z(t), z(0) = z_0$, from terminal set M for all allowed admissible controls $u = u(t) \in P, u(\cdot) \in U$ and $t \geq 0$. The goal of the pursuer is generally opposite.

Here the trajectory $z(t)$ of the system (1) is the solution of (1) corresponded to the selected controls. At any given moment $t \geq 0$ the evader may use the values $z(t)$ and pursuer's control $u(t)$ at the same moment, but does not know the control of the pursuer in future moments of time. Certainly, the evader knows system (1), P, Q and M.

2. Purpose of the work

1) Development of effectively verifiable sufficient conditions to ensure a solving of evasion problem in the classes of locally inertial controls.

2) Algorithms of design of locally inertial evasion control.

3) Demonstration of obtained results on example well known in differential games.

3. Short history of differential games and evasion problems in inertial controls

The statement of the evasion problem formulated above belongs to L.S.Pontrjagin and E.F.Miščhenko [1]. In that work first fundamental results were obtained and represented the proof of solve of evasion problem in linear differential games (1) with control functions selected from the classes of measurable functions.

The theory of differential games was born in the late 50-th of the last century, when the first articles on differential games began to appear in the open press ([2-10]). The founder of the theory of differential games is Rufus Isaacs, which issues in 1965 fundamental monograph

"Differential Games" [11]. The theory of differential games (this term belongs to R.Isaacs) is a natural generalization of the problems of the theory of optimal control, complicated by the presence of conscious opposition of rivals, as well as obstacles, uncertainties, etc.

Ideas of the theory of differential games rapidly spread around the world, but developed mostly by Soviet and American researchers and continue to develop in the Post-Soviet space. One of the first in the USSR was L.S.Pontrjagin, the creator of the Pontrjagin's Maximum Principle, an outstanding mathematician of the 20th century. On his initiative, the first book on differential games, published in 1965 by R. Isaacs [11], was translated to Russian in 1967, which may consider as extremely short period for that time.

Unlike the American mathematicians (including R.Isaacs) who studied and solved the differential game for two players at once or mainly pursuit problem, L.S. Pontrjagin divided the differential game into two separate problems: the problem of pursuit and the problem of evasion(or avoiding). He was the first to receive fundamental results for each of the problems [12-14]. In particularly, significant results were obtained by L.S. Pontrjagin on the evasion problem [12].

On his initiative, a lot of scientists were involved in the study of differential games. Independent scientific schools for the study of differential games were established in the of Moscow (L.S.Pontrjagin and E.F.Miščhenko), Ekaterinburg (N.N.Krasovsky), St. Petersburg (L.A.Petrosjan), Kiev (B.N.Pshenychniĭ and A.A.Chikriĭ) and Tashkent (N.Satimov), which are still functioning. Representatives of these schools received significant results in the wide directions. Because of too long list of published papers in differential games, we introduce to your attention very small part of them, including main papers and monographs concerning our investigations (see, for example, [15-25]).

4. Methods of researching

The theory of ordinary differential equations, functional analysis and linear algebra are used in researching evasion problem (1).

5. Main results in solving of evasion problem in locally inertial controls

5.1. Locally inertial controls

Definition 1. Let D – is the set of such continuous functions $u(t)$, $0 \le t < \infty$, with values from P, that for any $\varepsilon > 0$ there exists $\delta(\varepsilon) > 0$, such that $|u(t_1) - u(t_2)| \le \varepsilon$ for any $t_1, t_2 \in [0, \infty), |t_1 - t_2| \le \delta(\varepsilon)$.

Definition 2. $D(r, \triangle)$ is the set of all measurable functions $u(t) \in P$, $0 \le t \le \infty$, such that $|u(t_1) - u(t_2)| \le r$, for any $t_1, t_2 \in [0, \infty), |t_1 - t_2| \le \triangle$, where r and $\triangle$ are constants.

Classes of functions D and $D(r, \triangle)$ was introduced by N. Satimov in [26, §2] and called as classes of inertial functions, which was introduced for pursuer only.

Remark 1. The evasion problem (1) with inertial controls for pursuer and measurable controls for evader is considered and solved in [26].

Definition 3. $L_P(t_0, \triangle)$ is the class of measurable functions of pursuer $u = u(t) \in P$, $t \ge t_0$, such that in any interval $[t_0, t_0 + \triangle]$ holds

$A) u(t_i) = u_i \in P$;

$B)$ $|u(t) - u(t_i)| \le \gamma |t - t_i|^\alpha, t \in [t_i, t_{i+1}]$, where $i = 1, 2, \ldots, p, p \in N, t_i \in T = \{t_0 < t_1 < t_2 < \cdots < t_p = t_0 + \triangle\}$, T is any split of an interval $[t_0, t_0 + \triangle]$, $\gamma \ge 0, \alpha > 0$ and $\triangle > 0$ are constants. Similarly, class of controls $L_Q(t_0, \triangle)$ for evader is introduced.

Definition 4. $D_c(P)$ is the class of piecewise constant functions with values from P such that every function from it has on the length $[t_0, t_0 + \triangle], t_0 \ge 0, \triangle > 0$, finite number of first type discontinuity points.

Similarly the class of controls $D_c(Q)$ for evader is introduced.

Definition 5. Classes of controls $D_c(P), D_c(Q), L_P(t_0, \triangle)$ and $L_Q(t_0, \triangle)$ are called as locally inertial.

Definition 6. The positive real numbers, depending on known parameters of the game (1) only (C, P and Q, M, vector a *etc.*), and not depending on the controls we will call as constants.

Remark 2. Classes of functions $D_c(P) \subset L_P(t_0, \triangle)$ and $D_c(Q) \subset L_Q(t_0, \triangle)$.

5.2. Lemmas about zeroes of functions and maneuvering

First result on function's zeroes was obtained by L.S. Pontrjagin, who proved a lemma about zeroes for analytical functions [13, §4, п.A)]. Later that lemma named as L.S.Pontrjagin's lemma on zeroes.

N. Satimov significantly improved results of L.S. Pontrjagin by extending it on additional classes of non-analytical functions.

Lemma of N. Satimov (about zeroes of non-analytical functions) ([25, §1.3; [28]). Let is assume that $\varphi_i(\cdot), i = 1,2,3,\dots k,$ are real scalar functions well defined and analytical on interval of [0,1]; α_1 $,\alpha_2,\dots,\alpha_m$ and δ are non-negative numbers, Σ, Σ_1 and Σ_2 are linear finite dimensional sets of functions defined as follows:

$$\Sigma = \{\varphi(\cdot) : \varphi(t) = \sum_{i=1}^{k} c_i\varphi_i(t), 0 \le t \le 1, c_i \in R^1, \}$$

$$\Sigma_1 = \{\varphi(\cdot) : \varphi(t) = \sum_{i=1}^{k} t^{\delta} c_i\varphi_i(t) + c_{k+1}, 0 \le t \le 1, c_i \in R^1\}$$

$$\Sigma_2 = \{\lambda(\cdot) : \ \lambda(t) = \sum_{i=1}^{k} c_k\varphi_k(t) + \sum_{i=k+1}^{k+m} c_i t^{\alpha_i} \ , 0 \le t \le 1, c_i \in R^1,$$

$$i = 1,2,\dots,k+m\}$$

Let $N(f(\cdot))$ is a number of geometrically different zeroes of function $f(\cdot)$ on a closed interval $I \subset [0,1]$. Then there exists a natural number $\overline{N}$, dependent only on the sets of functions $\Sigma, \Sigma_1$and Σ_2 such that

$N(f(\cdot)) \le \overline{N}$, for any $f(\cdot) \in \Sigma \cup \Sigma_1 \cup \Sigma_2$.

Remark 2. Case $f(\cdot) \in \Sigma$ proved in [13, §4], and for $f(\cdot) \in \Sigma_1 \cup \Sigma_2$ in [25, §1.3]; [28]).

Let us formulate Lemmas on maneuvering (or on bypass maneuver). These lemmas are the base in differential evasion games for organizing the evasion process. They allow to show local evasion of moving point $z(t)$ from terminal set M on a closed interval $[0,\theta]$, where $\theta > 0$ is a constant. For the first time lemma on maneuvering was proved in [13, §4]. Different modifications of this lemma in two dimensional case are proved in [25, §1.3; 29-30], multidimensional case was considered in [31-32].

Let's introduce the following definitions:

$\widehat{\Sigma} = \left\{\varphi(\cdot) : \varphi(t) = \sum_{i=1}^{k} c_i\psi_i(t), c_i \in R^1, i = 1,2,\dots k\right\}$, $\psi_i(t)$ are analytical on $[0,1]$ functions and $\{\psi_i(t),\ i = 1,2,\dots k\ \}$ is a basis of $\widehat{\Sigma}$, $\left(\nu^1,\nu^2\right)$ is the orthogonal coordinate system in R^2; Γ is a square defined as $\left|\nu^i\right| \le \mu, i = 1,2, \mu > 0,$

$$\Omega(t) = \begin{pmatrix} d_1 t^{k_1} & d_3 t^{k_1+\beta} \\ d_2 t^{k_2} & d_4 t^{k_2+\beta} \end{pmatrix} \text{ is a } (2 \times 2) -\text{matrix},$$

$k_1$and k_2 are natural numbers, $0 \le k_1 \le k_2$, $\beta \in (\,0,1]$ и $d_1 d_4 - d_2 d_3 \ne 0$.

Lemma 1 on maneuvering ([25], §1.3, Lemma 1.3.3)

Let $\varphi(\cdot) = \left(\varphi^1(\cdot), \varphi^2(\cdot)\right)$ is any function with components from $\widehat{\Sigma}$. Then there exists a square $\Gamma^{'} \subset \Gamma$ with the side $\mu_1 > 0$, such that the function $\nu(t) = \varphi(t) - \Omega(t)\overline{\alpha}$, satisfies the inequality

$$|\nu(t)| \ge d_5 t^{k_2+\beta}, 0 \le t \le \theta_1, \tag{2}$$

for all $0 \le t \le \theta_1, \overline{\alpha} = (\alpha_1, \alpha_2) \in \Gamma^{'}$, where $\theta_1 > 0$ and $d_5 > 0$ are constants.

Lemma 2 on maneuvering ([25], §1.3, Lemma 1.3.4)

Let $\varphi(\cdot) = \left(\varphi^1(\cdot), \varphi^2(\cdot)\right)$ is any function, whose components are from $\widehat{\Sigma}$,

$h(\cdot) = \left(h^1(\cdot), h^2(\cdot)\right)$ is continuous on $[0,1]$ function, satisfying inequality

$$\left|h^j(t)\right| \le d_6 t^{\delta}, j = 1,2; t \in [0,1]; \delta \in (0,1] \text{ and } d_6 > 0 \text{ are constants.}$$

Then there exists a point $\alpha_0 = \left(\alpha_0^1, \alpha_0^2\right) \in \Gamma$ and a constant $\theta \in (0,1]$ such that for the function

$$p\left(t\right) = \varphi\left(t\right) - \Omega\left(t\right)\left[\alpha_0 + h\left(t\right)\right], 0 \leq t \leq 1,$$

an inequality

$$\left|p\left(t\right)\right| > d_7 t^{k_2+\beta}, 0 \leq t \leq \theta_2, \tag{3}$$

is fulfilled, where $d_7 > 0$ and θ_2 are constants.

Remark 4. Lemma 1 for the case $\beta = 0$ proved in ([13], §4).

A proof of Lemma 2 based on Lemma 1. In the Lemma 2 squares Γ and Γ' are the same as in Lemma1 on maneuvering, α_0 is the center of the square $\Gamma', \theta_2 = min\{1, \left(\frac{\mu_1}{2d_6}\right)^{\frac{1}{\delta}}\} > 0$ is a constant.

Remark 5. Lemmas about zeroes of function and maneuvering play important role in differential evasion games. They are allowing to organize local avoiding(evasion) process of trajectory from terminal set. Inequalities (2) and (3) are fundamental in evasion theory, because by its using the possibility of local evasion proved, extended to infinite time interval.

5.3. Sufficient Conditions and Theorem of Evasion

Let the differential evasion game (1) starts from $z\left(0\right) = z_0 \notin M$ at the moment $t_0 = 0$. Further let L is the orthogonal complement to M in R^n , W is any two dimensional subspace in L, π is orthogonal operator from R^n on W. We will assume that an orthogonal system (w_1, w_2) is introduced in R^2 and let $[w]_j, j = 1,2,$ are coordinates of vector $w \in R^2$ with respect to (w_1, w_2). Further let's denote $D : R^n \to R^2$ some linear mapping; $[D]_j$ *is* j-row of D, i.e. $[D]_j z = [Dz]_j, j = 1,2.$

Assumption 1. There exists $W \subset L$, linear mapping $F : W \to R^2$, natural numbers $k_1 \leq k_2$ such that

A) $\left[F\pi c^i P\right]_j = \left[F\pi c^i Q\right]_j = 0$ for all $i = 0,1,2,\ldots,k_j - 2;;j = 1,2;;$

B) The set $R = DQ$ comprises inner point (with respect to W), where

$D = \left(\left[F\pi c^{k_1-1}\right]_1, \left[F\pi c^{k_2-1}\right]_2\right)^T$ is a $2 \times n$ matrix, mapping R^non R^2, T is transposition operation.

Assumption 2. There exist $W \subset L$, linear mapping $F : W \to R^2$, natural numbers $k_1\ k_2, k_1 \neq k_2$, such that

A) $\left[F\pi c^i P\right]_j = \left[F\pi c^i Q\right]_j = 0, i = 0,1,2,\ldots,k_j - 2, j = 1,2.$

B) The set $R = DQ$ is an interval, which is not parallel to coordinate axis $[w]_1 = 0$ and $[w]_2 = 0$.

Theorem of evasion. If at least one of above mentioned assumptions is fulfilled then in (1) evasion is possible from any starting point $z_0 \in R^n \backslash M$ for any $t \geq 0$ in classes of locally inertial controls.

The proof of the theorem is split into two parts.

5.3.1 Proof of the Theorem of evasion(case of the Assumption 1)

Let in the evasion game (1) the Assumption 1 is fulfilled and $z_0 = z\left(0\right) \in R^n \backslash M$. Further, let origin $O \in R^n$ is an inner point of the set R (we can achieve this by moving inner point R with vector $a \in R^n$, included in equation (1)). For the sake of simplicity and without loss of generality we will set

$t_0 = 0, \triangle = 1, \alpha \in (0,1]$, then $D_P\left(t_0, \triangle\right) = D_P\left(0,1\right) = D_P$ and $D_Q\left(t_0, \triangle\right) = D_Q\left(0,1\right) \equiv D_Q$.

Let players choose their locally inertial controls $u\left(t\right) \in D_P$ and $v\left(t\right) \in D_Q, t \in [0,1]$ and $z_0 \in R^n \setminus M$. Then we will have from Cauchy formula [33, Chapter 3, §4] for the trajectory $z(t)$ next representation

$$\pi z(t) = \pi e^{tC} z_0 + \int_0^t \pi e^{(t-s)C}\left(u\left(s\right) + v\left(s\right) + a\right) ds. \tag{4}$$

Let

$$u\left(s\right)=\left(u\left(0\right)+\overline{u}_{0}\left(s\right)\right)\in D_{P},\qquad v\left(s\right)=v_{0}+\overline{v}_{0}\left(s\right)\in D_{Q},s\in\left[0,1\right],\tag{5}$$

where $u\left(0\right)=u_{0}\in P,v\left(0\right)=v_{0}\in Q$. Then functions $\overline{u}_{0}\left(s\right)$ and $\overline{v}_{0}\left(s\right)$ will satisfy the following inequalities:

$$\left|\overline{u}_{0}\left(s\right)\right|\leq\gamma s^{\alpha},\left|\overline{v}_{0}\left(s\right)\right|\leq\gamma s^{\alpha},s\in\left[0,1\right].$$

By inserting (5) in (4), we get:

$$\left[F\pi z\left(t\right)\right]_{j}=\left[F\pi e^{tC}z_{0}\right]_{j}+\int_{0}^{t}\left[F\pi e^{(t-s)C}\left(u_{0}+a\right)\right]_{j}ds+$$
$$+\int_{0}^{t}\left[F\pi e^{(t-s)C}\overline{u}_{0}\left(s\right)\right]_{j}ds+\int_{0}^{t}\left[F\pi e^{(t-s)C}(v_{0}+\overline{v}_{0}\left(s\right)\right]_{j}ds\tag{6}$$

where $j=1,2;t\in\left[0,1\right]$.

Let's estimate integrals in (6).

From condition A) of Assumption 1 and exponent expansion

$$e^{tC}=E+tC+\cdots+\frac{t^{k}}{k!}C^{k}+\ldots,$$

where $E-$ identity $(n\times n)-$ matrix, it is easy to get

$$\int_{0}^{t}\left[F\pi e^{(t-s)C}\overline{u}_{0}\left(s\right)\right]_{j}ds=\int_{0}^{t}\left[\sum_{i=k_{j}-1}^{\infty}\frac{(t-s)}{i!}\pi C^{i}\overline{u}_{0}\left(s\right)\right]_{j}ds\equiv h_{j}^{1}\left(t\right),\tag{7}$$

Moreover,

$$|h_{j}^{1}\left(t\right)|\leq d_{1}t^{k_{j}+\alpha},t\in\left[0,1\right],\tag{8}$$

where $\alpha\in\left(0,1\right]$ (see D_{P} or D_{Q}), $j=1,2$ and $d_{1}>0$ is a constant.

Like in (7), after simple transformations we get

$$\int_{0}^{t}\left[F\pi e^{(t-s)C}v_{0}\right]_{j}ds=$$
$$=\frac{t^{k_{j}}}{k_{j}!}\left[F\pi C^{k_{j}-1}v_{0}\right]_{j}+\int_{0}^{t}\sum_{i=k_{j}}^{\infty}\frac{(t-s)^{i}}{i!}\left[F\pi C^{i}v_{0}\right]_{j}ds=\frac{t^{k_{j}}}{k_{j}!}\left[F\pi C^{k_{i}-1}v_{0}\right]_{j}+h_{j}^{2}\left(t,v_{0}\right),\tag{9}$$

where

$$\left|h_{j}^{2}\left(t,v_{0}\right)\right|\leq d_{2}t^{k_{j}+\alpha},j=1,2;;t\in\left(0,1\right],d_{2}>0.\tag{10}$$

Further

$$\int_{0}^{t}\left[F\pi e^{(t-s)C}\overline{v}_{0}\left(s\right)\right]_{j}ds=\int_{0}^{t}\left[\sum_{i=k_{j}-1}^{\infty}\frac{(t-s)}{i!}\pi C^{i}\overline{v}_{0}\left(s\right)\right]_{j}ds=h_{j}^{3}\left(t\right),$$

and it is easy to get

$$\left|h_{j}^{3}\left(t\right)\right|\leq d_{3}t^{k_{j}+\alpha},j=1,2;;\alpha\in(0,1],t\in\left[0,1\right],\tag{11}$$

$d_{3}>0$ is a constant.

According to B) of Assumption 1 the set $R=DQ$ contains inner point and let l_{1} is one of them. Then, there exists a constant $\gamma_{1}>0$ and square $\Gamma_{1}=\{\nu\in R^{2}:\left|[\nu]_{j}\right|\leq\gamma_{1},j=1,2\}$ such that $l_{1}+\Gamma_{1}\subset R$.

Consider the equation with respect to v

$$Dv = l_1 + \omega, v \in Q, \omega \in \Gamma_1. \tag{12}$$

In general, there are more than one solutions for this equation, let us choose the smallest in lexicographic sense, it will uniquely define an algorithm for solving (12). We will set the smallest lexicographical solution of (12) as $v^* \in Q$, it will obviously depend on ω. Further, we offer to evader use special control $\tilde{v}(s) \equiv (v^* + \overline{v^*}(s)) \in D_Q$, $s \in [0,1]$ and $|\overline{v^*}(s)| \leq \gamma s^\alpha$, corresponding to given $\omega \in \Gamma_1$. Then inserting $\tilde{v}(s)$ in (6) and (9) instead of $v(s)$ we will get

$$[F\pi z(t)]_j = [\varphi(t, z_0, u_0)]_j + \frac{t^{k_j}}{k_j!}\left[\omega + \overline{h_1}(t)\right]_j, \tag{13}$$

where

$$[\varphi(z_0, t, u_0)]_j = \left[F\pi e^{tC} z_0\right]_j + \int_0^t \left[F\pi e^{(t-s)C}(u_0 + a)\right]_j ds + \frac{t^{k_j}}{j!}[l_1]_j, \tag{14}$$

$$[\overline{h_1}(t)]_j = \left(\frac{t^{k_j}}{k_j!}\right)^{-1}[h_j^1 + h_j^2(t) + h_j^3(t))].$$

It is easy to get from (8), (10), (11), that

$$\left|\left[\overline{h_1}(t)\right]_j\right| \leq d_4 t^\alpha, j = 1, 2;; d_4 > 0 - , \ t \in [0,1], \tag{15}$$

In vector form (13) will look like (since $\Gamma_1 = -\Gamma_1$) :

$$F\pi z(t) = \varphi_1(t, z_0, u_0) - \Omega_1(t)\left[\omega + \overline{h_1}(t)\right], 0 \leq t \leq 1, \tag{16}$$

where $\overline{h_1}(t) = ([\overline{h_1}(t)]_1, [\overline{h_1}(t)]_2)^T$, $\varphi_1(t, z_0, u_0) = (\varphi(t, z_0, u_0)_1, \varphi(t, z_0, u_0)_2)^T$, T is transposition and (2×2) matrix

$$\Omega_1(t) = \begin{pmatrix} \frac{t^{k_1}}{k_1!} & 0 \\ 0 & \frac{t^{k_2}}{k_2!} \end{pmatrix}. \tag{17}$$

Let us use to (16) the Lemma 2 on maneuvering. First for that it is necessary to choose in (16) suitable vector $\omega \in R$, and then control function of evader corresponding ω.

We can see that matrix e^{tC} is fundamental for the system

$$\dot{z} = Cz, \quad z(0) = z_0, \tag{18}$$

that's why all the solutions of (18) are linear combinations of solutions from fundamental system. For example, if the system of functions $\{\varphi_1(t), \ldots, \varphi_n(t)\}$ is the fundamental system of solutions (18) (columns of e^{tC} can be solutions), then

$$e^{tC} z_0 = \sum_{i=1}^n z_{0i}\varphi_i(t) \tag{19}$$

Further, each function $\varphi_i(t), i = 1, 2, \ldots, n$ is analytical. Similarly, we can infer the same for other terms in $\varphi(z_0, t, u_0)$ (see (14)). Thus, each component of $[\varphi(z_0, t, u_0)]_j$, $j = 1, 2$, of function $\varphi(z_0, t, u_0)$ is an element of a finite dimensional linear set of analytical functions. Let define it as $\sum$. Then we add to $\sum$ finite number of functions $t^{k_2 - k_1}\varphi_1(t), \ldots, t^{k_2 - k_1}\varphi_n(t)$ and t^{k_2}. The obtained set of functions will be also finite dimensional, let define it as $\sum_1$.

We will use now the Lemma 2 on maneuvering to the relation (16). First of all we set $\beta = 0$ (see (17)). Then put

$$\widehat{\Sigma} = \sum_1, \Gamma_1 = \Gamma, \Omega(t) = \Omega_1(t), \varphi(\cdot) = \varphi_1(\cdot, z_0, u_0), \ \ p(\cdot) = F\pi z(\cdot), \ h(\cdot) = \overline{h_1}(\cdot), \ \delta = \alpha.$$

We obtain fulfillment all conditions of Lemma 2, hence according to it there exist a vector $\omega = \omega_0 \in {}_1$ such that from (16) we get inequality

$$|F\pi z(t)| = \left|\varphi_1(t, z_0, u_0) - \Omega_1(t)\left[\omega_0 + \overline{h_1}(t)\right]\right| > d_7 t^{k_2} \tag{20}$$

for all $t \in (0, \theta_0]$, where $\theta_0 > 0$ and $d_7 > 0$ are constants.

From (20) one has the inequality

$$|\pi z(t)| \geq d_8 t^{k_2}, \quad t \in (0, \theta_0], \quad d_8 > 0 \text{ is a constant.} \tag{21}$$

Inequality (21) provides local avoiding trajectory $z(t)$ from terminal set M in time interval $[0, \theta_0]$ by using the control $\tilde{v}(t)$. If $t = 0$, then $|\pi z(0)| = |\pi z_0| > 0$, since $z_0 \in R^n \backslash M$, (π is the projection on orthogonal complement to M). If t $\in (0,\theta_0]$, then from (28) we get, that $|\pi z(t)| > 0$, which means z $(t) \notin$ M. What evasion control from D_Q will provide (21)? Now we will explane a way of chosing of this evasion control.

For a vector ω_0 we can determine correspondence vector $\overline{v_0} \in$ Q as a lexicographic solution of equation (12) with $\omega = \omega_0$. Then it is recommended to evader to use the admissible control function as $\tilde{v}_0(t) = \overline{v_0} + v_0(t) \in D_Q$ with $|v_0(t)| \leq \gamma t^{\alpha}, t \in [0,\theta_0]$. This control function $\tilde{v}_0(t)$, $t \in (0, \theta_0]$ depends on $u(0) = u_0$ too (see (20)).

The evader have to use $\widetilde{v}_0(t)$, $t \in (0, \theta_0]$ till the first moment t_1, when will $|u(t_1) - u_0| > \gamma {t_1}^{\alpha}$. From the moment t_1 pursuer uses some admissible control $u(t) \in D_P$ with known for the evader value $u(t_1) = u_1$only. If $t_1 \leq \theta_1$, then evader changes his control function by substitutions: t_1 to t_0, $z(t_1)$ to z_0, u_1 to u_0, ω_1 to ω_0 and builds corresponding control functio $\tilde{v}_1(t) = (\overline{v}_1 + v_1(t)) \in D_Q$ (received like $\tilde{v}_0(t)$), where $|v_1(t)| \leq \gamma t^{\alpha}$ and $\overline{v_1}$ is the lexicographic solution of equation (12), corresponding to ω_1.

After these substitutions we get inequality (like (21))

$$|\pi z(t)| > d_9 t^{\overline{k}}, t \in (t_1, t_1 + \theta_1], d_9 \text{ and } \theta_1 \text{ are constants,}$$

that provide evasion from M on $[t_1, t_1 + \theta_1]$. The pursuer may change his control function from D_P finite times only at [0,1], that is why the evader provides avoiding process on [0,1].Repeating this process step by step, we get avoiding $z(t)$ from terminal set for all $t \geq 0$.

5.4.3. Proof of the Theorem of evasion (case of the Assumption 2)

Let in the evasion differential game (1) the Assumption 2 is fulfilled. In this case, set R is a closed interval which is not parallel to axis of orthogonal coordinate system (w_1, w_2), introduced in R^2. Let l_2 is the center of the R, then coordinates $[\nu]_1$and $[\nu]_2$ of any point $\nu \in R - l_2$ satisfies the condition

$$[\nu]_2 = \lambda[\nu]_1, \lambda \neq 0, |[\nu]_1| \leq c_1, \tag{22}$$

where $|\lambda| > 0$ and $c_1$are constants.

Let us consider for given $\nu \in R - l_2$ an equation with respect to $v \in Q$:

$$Dv = l_2 + \nu. \tag{23}$$

For any $\nu \in R - l_2$ there exists at least one solution of (23).The smallest in lexicographical sense solution of equation (23) we will define as $v = v^* \in Q$, obviously, it depends on ν. Let $z(t)$ is the solution of system (1), corresponding to pursuer's control $u(t) = u_0 + \overline{u}(t) \in D_P$ and evader$^{'}$s control (we give the rul of choosing later)

$$\tilde{v}(t) = v_0 + \overline{v}_0(t) \in D_Q, \tag{24}$$

where $|\overline{u}(t)| \leq \gamma t^{\alpha}$ and $|\overline{v}_0(t)| \leq \gamma t^{\alpha}$, $t \in [0,1], \alpha \in (0,1]$.

Then, using conditions A) and B) of Assumption 2 and (23)-(24) we get

$$F\pi z(t) = \varphi_2(t, z, u_0) + \int_0^t \overline{\Omega}(t)\nu d\tau + \overline{h}_2(t), \tag{25}$$

where $\overline{\Omega} = \begin{pmatrix} \frac{\tau^{k_1}}{k_1!} & 0 \\ 0 & \frac{\tau^{k_2}}{k_2!} \end{pmatrix}, \nu = \begin{pmatrix} [\nu]_1 \\ \lambda[\nu]_1 \end{pmatrix};$

$$[\varphi_2(t,z_0,u_0)]_j=\left[F\pi e^{tC}z_0\right]_j+\int_0^t[F\pi e^s a]_j ds+[l_2]_j\frac{t^{k_j}}{k_i!};$$

$$[h_2(t)]_j=\int_0^t\{\sum_{i=k_j}^{\infty}\frac{(t-s)^i}{i!}\left[F\pi C^i\overline{v}_0(s)\right]_j\}ds$$

It is easy to see that functions $[h_2(t)]_j, j=1,2;;\ t\in[0,1]$, are continous and satisfy the eqation

$$\left|[h_2(t)]_j\right|\leq c_2t^{k_j+\alpha}, t\in[0,1], j=1,2.$$

Let $\Gamma_2=\left\{\nu\in R^2:\left|[\nu]_j\right|\leq\frac{c_2}{2}, j=1,2\right\}$ and let $\nu_0=([\nu_0]_1,[\nu_0]_2)^T$ is some point from Γ_2. Let set in (25) a vector $\nu=([\nu]_1\ [\nu]_2\ )^T$, T is transposition, as follows:

$$[\nu]_1=[\nu_0]_1+t^\alpha[\nu_0]_2, \tag{26}$$

$$[\nu]_2=\lambda([\nu_0]_1+t^\alpha[\nu_0]_2),\ 0\leq t\leq 1. \tag{27}$$

Then, from definition of Γ_2 for any $\nu_0\in\Gamma_2$ we have:

$$|[v_1]_1|=|[\nu_0]_1+t^\alpha[\nu_0]_2|\leq|[\nu_0]_1|+|[\nu_0]_2|\leq c_2,$$

for any $t\in[0,1]$.

Now let insert in (25) a functions (26) and (27), then we get:

$$[F\pi z(t)]_1=[\varphi_2(t,z_0,u_0)]_1+\lambda\int_0^t\frac{(t-s)^{k_1-1}}{(k_1-1)!}([\nu_0]_1+s^\alpha[\nu_0]_2)ds+[h_2(t)]_1, \tag{28}$$

$$[F\pi z(t)]_2=[\varphi_2(t,z_0,u_0)]_2+\lambda\int_0^t\frac{(t-s)^{k_2-1}}{(k_2-1)!}([\nu_0]_1+s^\alpha[\nu_0]_2)ds+[h_2(t)]_2. \tag{29}$$

Integral in the right part of (28) may compute directly by applying Newton-Leibnitz formula:

$$\int_0^t\frac{(t-s)^{k_2-1}}{(k_2-1)!}([\nu_0]_1+s^\alpha[\nu_0]_2)ds=[\nu_0]_1\frac{t^{k_1}}{k_1!}+[\nu_0]_2\frac{2^{k_1}k_1!}{(2k_1+1)!}t^{k_1+\alpha}$$

Integral in (29) is computed similarly.

After simple calculation we have, that

$$F\pi z(t)=\varphi_2(t,z_0,u_0)-\Omega_2(t)[\nu_0+h_3(t)], 0\leq t\leq 1, \tag{30}$$

where

$$\Omega_2(t)=\begin{pmatrix}\frac{t^{k_1}}{k_1!} & \frac{2^{2k_1}k_1!}{(2k_1+1)!}t^{k_1+\alpha}\\ \lambda\frac{t^{k_2}}{k_2!} & \frac{\lambda 2^{2k_2}k_2!}{(2k_2+1)!}t^{k_2+\alpha}\end{pmatrix},$$

and $h_3(t)$ is continuous function on [0,1] satisfying inequality

$$|h_3(t)|\leq c_3t^\alpha, 0\leq t\leq 1, c_3>0 \text{ is a constant.}$$

We can prove, like in [25, §4], that if $k_1\neq k_2$, then $\det\Omega_2(\mathrm{t})\neq 0$ for any $t\in(0,1]$.

Let us show that above mentioned results give the possibility to evader use Lemma 2 on maneuvering. Each component of a function $\varphi_2(t,z_0,u_0)$ is an element of a linear finite dimensional family of analytical functions Σ_2. Now we can apply to the equation (30) the Lemma 2 by setting there

$$\widehat{\Sigma}=\Sigma_2,\ ={}_2,\quad \Omega(t)=\Omega_2(t),\ \varphi(\cdot)=\varphi_2(\cdot,z_0,u_0),\quad v(\cdot)=F\pi z(\cdot),$$

$$h(.)=h_3(\cdot),\quad \delta=\alpha\ ,\ \nu_0=\alpha_0(\text{center of square, giving by Lemma 2}).$$

$$|\pi z(t)| > c_4 t^{\overline{k}}, \tag{31}$$

where $\overline{k} = max\{k_1, k_2\} + \alpha$.

$t \in (0, \theta_0]$, $\theta_0 > 0$ and $c_4 > 0$ are constants.

Now, we will give a way of building of special evasion control, providing local evasion from terminal set. For a vector ν_0 from (26) we can determine corresponding vector $\overline{v_0} \in Q$ as a solution of equation (23). Then it is recommended to evader to use the admissible special control function

$\tilde{v}(t) = (\overline{v_0} + \overline{v_0}(t)) \in D_Q$ with $|\overline{v_0}(t)| \leq \gamma t^{\alpha}, t \in (0, \theta_0]$.

Inequality (31) provides local avoiding trajectory $z(t)$ from terminal set M in time interval $[0, \theta_0]$ by using the control $\tilde{v}(t)$. If $t = 0$, then $|\pi z(0)| = |\pi z_0| > 0$, since $z_0 \in R^n \backslash M$, (π is the projection on orthogonal complement to M). If $t \in (0, \theta_0]$, then from (31) we get, that $|\pi z(t)| > 0$, which means $z(t) \notin M$.

Inequality (31) is obtained with assumption that

$$u(t) = (u(0) + \overline{u}_0(t)) \in D_P, \quad \tilde{v}(t) = \overline{v_0} + \overline{v_0}(t) \in D_Q, \; t \in [0, \theta_0],$$

If at some moment $t_1 > 0$ (which is not known to the evader), pursuer decides to change control $u(t)$ on $\tilde{u}(t)$ for $t \geq t_1$ and considering $\tilde{u}(t) = u(t_1) + \overline{u}_0(t) \in D_Q$, where $|\overline{u}_0(t)| \leq \gamma t^{\alpha}$. In that case we make substitutions: t_1 to $t_0, z(t_1)$ to $z_0, \overline{v_1}$ to $\overline{v_0}$, ν_1 to ν_0 and setting $\tilde{v}(t) = \overline{v_1} + \overline{v_1}(t) \in D_Q$, where $|\overline{v_1}(t)| \leq \gamma t^{\alpha}$ is any function and $\overline{v_1}$ is the solution of equation (23), corresponding to ν_1(see (25)).

After these substitutions we get inequality

$$|\pi z(t)| > c_5 t^{\underline{k}}, t \in [t_1, t_1 + \theta_1]$$

providing evasion from M on $[t_1, t_1 + \theta_1]$.

Due to finite number of changes of pursuer's control $u(t)$ of on the segment [0,1], avoiding of trajectory of the system (1) is possible firstly on [0,1] and then step by step for all $t \geq 0$. Theorem of evasion is proved.

6. Discussion

6.1. About $\mu-$ problem of L.S.Pontrjagin

In the fundamental work [13] the evasion problem was solved by L.S. Pontrjagin when the pursuer had some geometric advantage in control over the evasion player(evader). He formulated evasion conditions in one of the following form [13,§1]:

1. Rotation condition

There is no one dimensional linear subspace $W^1 \subset L$ such that

$$\pi e^{tC} Q \subset W^1 \text{ for any small } t > 0.$$

2. Advantage condition

There exists a constant $\mu > 1$ such that

$$\mu e^{\pi t C} P \subset \pi e^{tC} Q \text{ for any small } t > 0.$$

The Advantage condition contains a constant $\mu > 1$. In 1977 the $\mu-$ problem of L.S. Pontrjagin was formulated so: is it possible to solve the evasion problem (1) by $\mu = 1$? A number of researchers started in order to solve the $\mu-$ problem of L.S. Pontrjagin in general case, when players use measurable control functions. But it was shown later, that there were examples when the problem of evasion was solved with $\mu = 1$ and on the other hand there were also counter examples.

Some of researchers tried to restrict the possibilities of the pursuer by offering him to use controls from narrower classes of functions, while the evader could use controls from more wide classes of measurable functions. With such restrictions Pontrjagin's μ - problem (evasion with $\mu = 1$) was solved in [26-27], where pursuer used inertial controls introduced for the first time by N.Satimov in [26] and approximately same controls by M.S.Nikolskiĭ in [27] for special differential game. The problem of evasion was solved in [26] for all $\mu > 0$ (including case $\mu = 1$).

In this paper for both players the classes of locally inertial controls are introduced, different from inertial controls [26-27], and in these classes the evasion problem (1), in particular $\mu-$ problem, is investigated and solved for all $\mu \geq 0$(including $\mu = 1$) .

6.2. Example

In the theory of differential evasion games the main checking example is well known Control Example of L.S.Pontrjagin [12,§6]. Let us set an evasion problem for it.

Motions of point x (pursuer) and point y (evader) is described by equations

$$\ddot{x} + \alpha\dot{x} = \rho u, x \in R^{\nu}, u \in R^{\nu}, P = \{u : |u| \leq 1\}, \rho \geq 0,\ \alpha \geq 0,$$

$$\ddot{y} + \beta\dot{y} = \sigma v, y \in R^{\nu}, v \in R^{\nu}, Q = \{v : |v| \leq 1\}, \sigma \geq 0,\ \beta \geq 0\}.$$

After introducing new variables

$$z_1 = x - y; z_2 = \dot{x};;\ z_3 = \dot{y};;\ z = (z_1, z_2, z_3)^T \in R^{3v}.$$

we get equations of linear differential game (see (1)):

$$\begin{cases} \dot{z}_1 = z_2 - z_3, \\ \dot{z}_2 = -\alpha z_2 + \rho u, \\ \dot{z}_3 = -\beta z_3 + \sigma v, \end{cases} \tag{32}$$

Terminal set is

$$M = \{z \in R^{3v} : z_1 = 0\}.$$

It's easy to check that Assumption 1and 2 of the Theorem of evasion is fulfilled if

$$1)\nu \geq 2 \text{ and } \sigma > 0\,(\text{Assumption } 1)\,, \tag{33}$$

2) $\nu \geq 2$ and Q contains in some coordinate system $(\nu_1, \nu_2) \in R^2 \subset R^{\nu}$. an interval, not parallel to axis

$$\nu_1 = 0 \textit{ and } \nu_2 = 0 \text{ (Assumption 2)}. \tag{34}$$

As we can see evasion conditions (33) or (34) do not depend on P , hence they do not have an influence on ability of evasion of object y from object x, if players use locally inertial controls.

In both cases (33) and (34) the absence of influence of P over Q tell us that $\mu-$ problem of L.S.Pontrjagin is solvable for $\nu \geq 2$ and any $\mu > 0$ (including $\mu = 1$) .

The conditions $\nu \geq 2$ and $\mu > 0$ are not enough for solving evasion game (32) by using of sufficient conditions and methods represented in [1;2;23-27;29-31;34;35].

7. Conclusion

In this work, evasion problem in linear differential games is considered. New classes of control function, named locally inertial controls for pursuer and evader are introduced. In these classes, which are used by both players, sufficient conditions for solving of evasion problem are represented and corresponding theorem of evasion is proved. Conditions of evasion does not contain any advantage condition of evader' s control parameters over pursuer's (in some geometrical since). This fact allows to solve, in particulary, well-known $\mu -$ problem of L.S. Pontrjagin in the classes of locally inertial controls. Results illustrated by example.

Part of results of this paper was announced in [36] (formulation of Assumption 1 and definitions of locally inertial controls).

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Локалді-инерциялық басқарумен берілген сызықты дифференциалдық қашу ойыны

Аннотация: Қуушы және қашушы екі ойыншысы бар сызықты дифференциалдық ойындарда қашу есебі қарастырылады. Локалді-инерциялық деп аталатын ойыншыларды басқарудың жаңа кластары енгізіледі. Бұл класстарда терминалды жиында жатпайтын кез келген бастапқы позициядан бастап қашу есебі қарастырылады. Қашудың тиімді тексерілетін жеткілікті шарттары алынып, қашушы ойыншыны басқарудың алгоритмдері құрылды. Алынған нәтижелердің артықшылығы дифференциалдық ойындар теориясында белгілі мысалдарында көрсетілді.

Түйін сөздер: қашу, қуушы, қашушы ойыншы, локалді-инерциялық басқару, басқару параметрлері, терминалды жиын.

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Линейная дифференциальная игра убегания с локально - инерционными управлениями

Аннотация: Рассматривается задача убегания в линейных дифференциальных играх с двумя игроками, преследователем и убегающим. Вводятся новые классы управлений игроков, называемые локально-инерционными. В этих классах управлений решается задача убегания из любой начальной позиции, не принадлежащей терминальному множеству. Получены эффективно проверяемые достаточные условия убегания и указаны алгоритмы построения управления убегающего игрока. Преимущества полученных результатов показаны на известном теории дифференциальных игр примере.

Ключевые слова: убегание, преследователь, убегающий игрок, управление, локально-инерционное управление, управляющие параметры, терминальное множество.

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«Л.Н. Гумилев атындағы Еуразия ұлттық университетінің хабаршысы. Математика. Компьютерлік ғылымдар. Механика сериясы» журналына жіберілетін жұмыстарға қойылатын талаптар

Журнал редакциясы авторларға осы нұсқаулықпен толық танысып, журналға мақала әзірлеу мен дайын мақаланы журналға жіберу кезінде басшылыққа алуды ұсынады. Бұл нұсқаулық талаптарының орындалмауы сіздің мақалаңыздың жариялануын кідіртеді.

1. Автордың қолжазбаны редакцияға жіберуі мақала авторының басып шығарушы, Л.Н. Гумилев атындағы Еуразия ұлттық университетіне мақаласын басуға келісімін және кез келген шетел тіліне аударылып қайта басылуына келісімін білдіреді.

2. Баспаға (барлық жариялаушы авторлардың қол қойылған қағаз нұсқасы және электронды нұсқасында) журналдың түпнұсқалы стильдік файлының міндетті қолданысымен LaTeX баспа жүйесінде дайындалған Tex- пен Pdf-файлындағы жұмыстар ұсынылады. Стильдік файлды ***bulmathmc.enu.kz*** журнал сайтынан жүктеп алуға болады.

3. Мақаланың көлемі 6 беттен кем және 18 беттен артық болмауы тиіс. Талап деңгейінен асқан жұмыстар редакциялық алқа отырысында қаралып, баспаға ерекше жағдайда ғана рұқсат етіледі.

4. Жұмыстың мәтіні ХҒТАР (Халықаралық ғылыми-техникалық ақпарат рубрикаторы) кодының көрсеткішімен басталып, кейін автор(лар)дың аты және тегі, жұмыс орнының толық атауы, қаласы, мемлекеті, E-mail-ы, мақаланың толық атауы, аннотациясы көрсетіледі. Аннотация 150-200 сөз көлемінде болуы тиіс, сонымен қатар мәтінде күрделі есептік формулалар болмауы, мақаланың толық аты қайталанбауы, жұмыстың мәтіні мен әдебиеттер тізімінде көрсетілетін сілтемелер болмауы керек. Аннотация мақаланың ерекшеліктерін көрсететін және оның құрылымын (кіріспе, есептің қойылымы, мақсаты, тарихы, зерттеу әдістері, нәтижелер және олардың талқылаулары, қорытынды) сақтайтын мақаланың қысқаша мазмұны болуы тиіс.

Журналдың потенциалды авторлары мақала құрылымы бойынша келесі талаптарды ұстанулары қажет:

- Мақала мәтінін түсінуді қамтамасыз ететін қажетті белгілер мен анықтамалар;
- Мақалада қарастырылатын есептің қойылымы;
- Қарастырылатын есеп бойынша тарихи мәліметтер - мақала тақырыбына сәйкес бұрын алынған нәтижелер кіммен және қашан алынғандығы туралы толық сілтемелерімен берілген ақпарат;
- Кез келген ғылыми жұмыстың ең жауапты бөлігі ретінде мақаланың қажеттілігі мен өзектілігін негіздеу;
- Мақалада қойылған есеп шешімін нақты тұжырымдау және сипаттау;
- Бұрын белгілі мәнмәтінінде мақала нәтижесінің(нәтижелерінің) жаңалы?ын егжей-тегжейлі негіздеу;
- Есептің шешімі толық негіздеулермен (дәлелдемелермен) жабдықталуы тиіс.

Осы талаптардың ең болмағанда біреуі сақталмаған жағдайда мақала қарастыруға қабылданбайды.

5. Жұмыстың мәтінінде кездесетін таблицалар мәтіннің ішінде жеке нөмірленіп, мәтін көлемінде сілтемелер түрінде көрсетілуі керек. Суреттер мен графиктер PS, PDF, TIFF, GIF, JPEG, BMP, PCX форматындағы стандарттарға сай болуы керек. Нүктелік суреттер кеңейтілімі 600 dpi кем болмауы қажет. Суреттердің барлығы да айқын әрі нақты болуы керек.

6. Жұмыста қолданылған әдебиеттер тек жұмыста сілтеме жасалған түпнұсқалық көрсеткішке сай (сілтеме беру тәртібінде немесе ағылшын әліпбиі тәртібі негізінде толтырылады) болуы керек. Баспадан шықпаған жұмыстарға сілтеме жасауға тиым салынады.

Сілтемені беруде автор қолданған әдебиеттің бетінің нөмірін көрсетпей, келесі нұсқаға сүйеніңіз дұрыс: тараудың номері, бөлімнің номері, тармақтың номері, теораманың номері (лемма, ескерту, формуланың және т.б.) номері көрсетіледі. Мысалы: «... қараңыз . [3; § 7, лемма 6]», «...қараңыз [2; 5 теорамадағы ескерту]». Бұл талап орындалмаған жағдайда мақаланы ағылшын тіліне аударғанда сілтемелерде қателіктер туындауы мүмкін.

Қолданылаған әдебиеттер тізімін рәсімдеу мысалдары

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7. Әдебиеттер тізімінен соң автор өзінің библиографикалық мәліметтерін орыс және ағылшын тілінде (егер мақала қазақ тілінде орындалса), қазақ және ағылшын тілінде (егер мақала орыс тілінде орындалса), орыс және қазақ тілінде (егер мақала ағылшын тілінде орындалса) жазу қажет. Соңынан транслиттік аударма мен ағылшын тілінде берілген әдебиеттер тізімінен соң әр автордың жеке мәліметтері (қазақ, орыс, ағылшын тілдерінде – ғылыми атағы, қызметтік мекенжайы, телефоны, e-mail-ы) беріледі.

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