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High-dimensional Collocation Weighted Approximations For Parametric Elliptic PDEs With Lognormal Inputs

Abstract: We constructed linear non-adaptive methods of non-fully and fully discrete polynomial interpolation weighted approximation for parametric and stochastic elliptic PDEs with lognormal inputs and proved convergence rates of the approximations by them. Our methods are sparse-grid collocation methods. Moreover, the fully discrete methods can be seen as multilevel methods of approximation. The Smolyak sparse interpolation grids in the parametric domain are constructed from the roots of Hermite polynomials or their improved modifications.

Key words: High-dimensional approximation, parametric and stochastic elliptic PDEs, lognormal inputs, collocation method, non-adaptive weighted polynomial interpolation approximation.

AMS Mathematics Subject Classificstion: 65C30, 65D05, 65D32, 65N15, 65N30, 65N35.

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1. Introduction

Let $D \subset \mathbb{R}^d$ be a bounded Lipschitz domain. Consider the diffusion elliptic equation

$$-\operatorname{div}(a\nabla u) \;=\; f \quad \text{in} \quad D, \quad u|_{\partial D} \;=\; 0, \tag{1}$$

for a given fixed right-hand side f and spatially variable scalar diffusion coefficient a. Denote $V := H^1_0(D)$ - the energy space and $V' := H^{-1}(D)$. If a satisfies the ellipticity assumption

$$0 < a_{\min} \le a \le a_{\max} < \infty,$$

by the well-known Lax-Milgram lemma, for any $f \in V'$, there exists a unique solution $u \in V$ in weak form which satisfies the variational equation

$$\int_D a\nabla u \cdot \nabla v \, \mathrm{d}\boldsymbol{x} \;=\; \langle f, v\rangle, \quad \forall v \in V.$$

We consider diffusion coefficients having a parametrized form $a = a(\boldsymbol{y})$, where $\boldsymbol{y} = (y_j)_{j=1}^m$ is a sequence of real-valued parameters ranging in the space $\mathbb{R}^m$. Here, the parametric dimension m may be arbitrarily high. The resulting solution to parametric and stochastic elliptic PDEs map $\boldsymbol{y} \mapsto u(\boldsymbol{y})$ acts from $\mathbb{R}^m$ to the space V. The objective is to achieve numerical approximation of this complex map by a small number of parameters with some guaranteed error in a given norm. Depending on the nature of the modeled object, the parameter $\boldsymbol{y}$ may be either deterministic or random. In the present paper, we consider the so-called lognormal case when the diffusion coefficient a is of the form

$$a(\boldsymbol{y}) = \exp(b(\boldsymbol{y})), \quad b(\boldsymbol{y}) = \sum_{j=1}^m y_j \psi_j, \tag{2}$$

where the y_j are i.i.d. standard Gaussian random variables and $\psi_j \in L_\infty(D)$. Parametric PDEs are of great interest in Uncertainty Quantification for modelling many complex phenomena involving high-dimensional or infinite-dimensional parameters which may be deterministic or stochastic. We refer the reader to [3, 4, 9, 13, 14] and references there for different aspects in approximation for parametric and stochastic PDEs.

In order to study fully discrete approximations of the solution $u(\boldsymbol{y})$ to the parametrized elliptic PDEs (1), we assume that $f \in L_2(D)$ and $a(\boldsymbol{y}) \in W^1_\infty(D)$, and hence we obtain that $u(\boldsymbol{y})$ has the second higher regularity, i. e., $u(\boldsymbol{y}) \in W$ where W is the space

$$W := \{v \in V \ : \ \Delta v \in L^2(D)\}.$$

equipped with the norm

$$\|v\|_W := \|\Delta v\|_{L^2(D)},$$

which coincides with the Sobolev space $V \cap H^2(D)$ with equivalent norms if the domain D has $C^{1,1}$ smoothness. Moreover, we assume that there holds the following *approximation property* for the spaces V and W: there are a sequence $(V_n)_{n=0}^\infty$ of linear subspaces of V with dimension $\le n$, a sequence $(P_n)_{n=0}^\infty$ of linear operators from V into V_n, a constant $C > 0$ and a number $\alpha > 0$ such that

$$\|P_n(v)\|_V \le C, \quad \|v - P_n(v)\|_V \le C\, n^{-\alpha} \|v\|_W, \quad n \in \mathbb{N}_0, \quad v \in W. \tag{3}$$

Based on spatial and parametric aproximability, namely, the approximation property (3) in the spatial domain and weighted ℓ_2-summability of the V and W norms of Hermite polynomial expansion coefficients obtained in [1,2], we explicitly constructed linear non-adaptive methods of non-fully and fully discrete polynomial interpolation approximation for parametric and stochastic elliptic PDEs with lognormal inputs (2), and proved corresponding convergence rates of the approximations by them. The linear non-adaptive methods of non-fully and fully discrete polynomial interpolation approximation are sparse-grid collocation methods. The Smolyak sparse grids in the parametric domain are constructed from the roots of Hermite polynomials or their improved modifications. Moreover, the fully discrete polynomial interpolation approximation can be represented in the form of a multilevel approximation.

Let X be a Hilbert space, $\mu(\boldsymbol{y})$ a positive measure on $\mathbb{R}^m$ and $0 < p \le \infty$. The $\mu(\boldsymbol{y})$ induces the Bochner space $L_p(\mathbb{R}^m, X, \mu)$ of μ-measurable mappings v from $\mathbb{R}^m$ to X equipped with the (quasi-)norm

$$\|v\|_{L_p(\mathbb{R}^m, X, \mu)} := \left(\int_{\mathbb{R}^m} \|v(\cdot, \boldsymbol{y})\|_X^p \, \mathrm{d}\mu(\boldsymbol{y}) \right)^{1/p},$$

with the change to ess sup norm with regard to $\mu(\boldsymbol{y})$, when $p = \infty$. We make use the abbreviation: $L_p(\mathbb{R}^m, X) := L_p(\mathbb{R}^m, X, \mu)$ if $\mu(\boldsymbol{y})$ is the Lebesgue measure on $\mathbb{R}^m$.

Let $\gamma(y)$ be the probability measrure on $\mathbb{R}$ with the standard Gaussian density:

$$\mathrm{d}\gamma(y) := g(y)\,\mathrm{d}y, \quad g(y) := \frac{1}{\sqrt{2\pi}} e^{-y^2/2}.$$

With some abuse we define tensor product measure $\gamma(\boldsymbol{y})$ on $\mathbb{R}^m$ as

$$\mathrm{d}\gamma(\boldsymbol{y}) := \bigotimes_{j=1}^m g(y_j)\mathrm{d}y_j \ = \ g(\boldsymbol{y})\mathrm{d}\boldsymbol{y}, \quad g(\boldsymbol{y}) := \bigotimes_{j=1}^m g(y_j).$$

Notice that $u(\boldsymbol{y})$ belongs to the space $L_p(\mathbb{R}^m, V, \gamma)$ for $0 < p < \infty$ [1], and there holds the expansion by the Hermite series

$$u(\boldsymbol{y}) = \sum_{\boldsymbol{s} \in \mathbb{N}_0^m} u_{\boldsymbol{s}} \, H_{\boldsymbol{s}}(\boldsymbol{y}), \quad u_{\boldsymbol{s}} \in V$$

converging in $L_2(\mathbb{R}^m, V, \gamma)$, where

$$H_{\boldsymbol{s}}(\boldsymbol{y}) = \bigotimes_{j \in \mathbb{N}} H_{s_j}(y_j), \quad u_{\boldsymbol{s}} := \int_{\mathbb{R}^m} u(\boldsymbol{y}) \, H_{\boldsymbol{s}}(\boldsymbol{y}) \, \mathrm{d}\gamma(\boldsymbol{y}), \quad \boldsymbol{s} \in \mathbb{N}_0^m,$$

and $(H_k)_{k=0}^{\infty}$ is the system of univariate orthonormal Hermite polynomials.

2. Lagrange interpolation in parametric domain

For every $n \in \mathbb{N}_0$, let $Y_n = (y_{n;k})_{k=0}^n$ be a sequence of points in $\mathbb{R}$ such that

$$-\infty < y_{n;0} < \cdots < y_{n;n-1} < y_{n;n} < +\infty; \quad y_{0;0} = 0.$$

For a function v on $\mathbb{R}$ taking values in a Banach space X, we define $I_n(v)$ for $n \in \mathbb{N}_0$ by

$$I_n(v) := \sum_{k=0}^{n} v(y_{n;k})\ell_{n;k}, \quad \ell_{n;k}(y) := \prod_{j=0,\ j\neq k}^{n} \frac{y - y_{n;j}}{y_{n;k} - y_{n;j}},$$

as the unique Lagrange polynomial interpolating v at $y_{n;k}$. Notice that $I_n(v)$ is polynomial of degree $\leq n$ and $I_n(\varphi) = \varphi$ for every polynomial φ of degree $\leq n$.

Let

$$\lambda_n(Y_n) := \sup_{\|v\sqrt{g}\|_{L_\infty(\mathbb{R})} \leq 1} \|I_n(v)\sqrt{g}\|_{L_\infty(\mathbb{R})}$$

be the Lebesgue constant. We want to choose sequences Y_n so that for some positive numbers τ and C, there holds the inequlity

$$\lambda_n(Y_n) \ \leq \ (Cn+1)^\tau, \quad \forall n \in \mathbb{N}_0. \tag{4}$$

We present two examples of Y_n satisfying (4).

The first example is the strictly increasing sequence $Y_n^* = (y_{n;k}^*)_{k=0}^n$ of the roots of H_{n+1}. Indeed, it was proven by Matjila and Szabados [11, 12, 15] that

$$\lambda_n(Y_n^*) \ \leq \ C(n+1)^{1/6}, \quad n \in \mathbb{N},$$

for some positive constant C independent of n (with the obvious inequality $\lambda_0(Y_0^*) \leq 1$). Hence, for every $\varepsilon > 0$, there exists a positive constant C_ε independent of n such that

$$\lambda_n(Y_n^*) \ \leq \ (C_\varepsilon n + 1)^{1/6+\varepsilon}, \quad \forall n \in \mathbb{N}_0. \tag{5}$$

The inequality (5) can be improved by the “method of adding points" suggested by Szabados [15] (for details, see also [10, Section 11]). More precisely, for $n > 2$, he added to Y_{n-2}^* two points $\pm\xi_{n-1}$, near $\pm a_{n-1}(g)$, which are given by the condition $|H_{n-1}\sqrt{g}|(\xi_{n-1}) = \|H_{n-1}\sqrt{g}\|_{L_\infty(\mathbb{R})}$. By this way, he obtained the sequence $\bar{Y}_n^* := \{-\xi_n, y_{n-2;0}, \cdots, y_{n-2,n-2}, \xi_n\}$ satisfying the inequality

$$\lambda_n(\bar{Y}_n^*) \ \leq \ C\log(n-1) \quad (n > 2)$$

which yields that for every $\varepsilon > 0$, there exists a positive constant C_ε independent of n such that

$$\lambda_n(\bar{Y}_n^*) \ \leq \ (C_\varepsilon n + 1)^\varepsilon, \quad \forall n \in \mathbb{N}_0.$$

For a given sequence $(Y_n)_{n=0}^\infty$, we define the univariate operator Δ_n^{I} for $n \in \mathbb{N}_0$ by

$$\Delta_n^{\mathrm{I}} := \ I_n - I_{n-1},$$

with the convention $I_{-1} = 0$. We introduce the operator $\Delta_{\boldsymbol{s}}^{\mathrm{I}}$ for $\boldsymbol{s} \in \mathbb{N}_0^m$ by

$$\Delta_{\boldsymbol{s}}^{\mathrm{I}}(v) := \bigotimes_{j\in\mathbb{N}} \Delta_{s_j}^{\mathrm{I}}(v)$$

for functions v defined on $\mathbb{R}^m$ taking values in the space V, where the univariate operator $\Delta_{s_j}^{\mathrm{I}}$ is applied to the univariate function v by considering v as a function of variable y_i with the other variables held fixed.

3. Collocation weighted approximations

For convenience, we introduce the convention $W^1 := V$ and $W^2 := W$. Our constructions of non-fully and fully discrete approximations are based on the approproperty (3) and the weighted ℓ_2-summability of the series $(\|u_{\boldsymbol{s}}\|_{W^r})_{\boldsymbol{s}\in\mathbb{N}_0^m}$, $r = 1, 2$, in the following lemma which has been proven in [1] for $r = 1$ and in [2] for $r = 2$.

Lemma 1. *Let* $r = 1, 2$ *. Assume that the right side* f *in* (1) *belongs to* $H^{r-2}(D)$ *, that the domain* D *has* $C^{r-2,1}$ *smoothness, that all functions* ψ_j *belong to* $W^{r-1,\infty}(D)$ *. Assume that there exist a number* $0 < q_r < \infty$ *and a sequence* $\boldsymbol{\rho_r} = (\rho_{r;j})_{j\in\mathbb{N}}$ *of positive numbers such that* $(\rho_{r;j}^{-1})_{j\in\mathbb{N}} \in \ell_{q_r}(\mathbb{N})$ *and*

$$\left\|\sum_{j=1}^{m} \rho_{r;j}|\psi_j|\right\|_{L^\infty(D)} + \sup_{|\alpha|\le r-1} \left\|\sum_{j=1}^{m} \rho_{r;j}|D^\alpha\psi_j|\right\|_{L^\infty(D)} < \infty \,.$$

Then for any $\eta \in \mathbb{N}$ *, there exists a constant* $C < \infty$ *independent of* m *such that*

$$\sum_{\boldsymbol{s}\in\mathbb{N}_0^m} (\sigma_{r;\boldsymbol{s}}\|u_{\boldsymbol{s}}\|_{W^r})^2 \le C \quad \textit{with} \quad \sigma_{r;\boldsymbol{s}}^2 := \sum_{\|\boldsymbol{s}'\|_{\ell_\infty(\mathbb{N}_0^m)}\le\eta} \binom{\boldsymbol{s}}{\boldsymbol{s}'} \boldsymbol{\rho}_r^{2\boldsymbol{s}'}.$$

We introduce the non-fully discrete polynomial interpolation operator I_Λ for a finite set $\Lambda \subset \mathbb{N}_0^m$ by

$$I_\Lambda := \sum_{\boldsymbol{s}\in\Lambda} \Delta_{\boldsymbol{s}}^{\mathrm{I}}.$$

Denote by $|G|$ the cardinality of a set G, and by $\Gamma(\Lambda)$ the grid of interpolation points in the operator I_Λ .

Theorem 1. *Let the assumptions of Lemma 1 hold for the spaces* $W^1 = V$ *with some* $0 < q_1 < 2$ *. Assume that* $(Y_n)_{n\in\mathbb{N}_0}$ *is a sequence such that every* Y_n *satisfies the condition* (4) *for some positive numbers* τ *and* C *. Define for* $\xi > 0$

$$\Lambda(\xi) := \{\boldsymbol{s} \in \mathbb{N}_0^m : \sigma_{1;\boldsymbol{s}}^{q_1} \le \xi\}.$$

Then for each $n \in \mathbb{N}$ *there exists a number* ξ_n *such that* $|\Gamma(\Lambda(\xi_n))| \le n$ *and*

$$\| \left(u - I_{\Lambda(\xi_n)}u\right) \sqrt{g}\|_{L_\infty(\mathbb{R}^m,V)} \le Cn^{-(1/q_1-1/2)}. \tag{6}$$

The constants C *in* (6) *is independent of* u *,* m *and* n *.*

For a finite subset G in $\mathbb{N}_0 \times \mathbb{N}_0^m$, denote by $\mathcal{V}(G)$ the subspace in $L_2(\mathbb{R}^m, V, \gamma)$ of all functions v of the form

$$v \; = \sum_{(k,\boldsymbol{s})\in G} v_k\, H_{\boldsymbol{s}}, \quad v_k \in V_{2^k}.$$

Let the approximation property (3) hold for the spaces V and W . For $k \in \mathbb{N}_0$ and $v \in W$, we define

$$\delta_k(v) := \; P_{2^k}(v) - P_{2^{k-1}}(v), \; k \in \mathbb{N}, \quad \delta_0(v) = P_0(v).$$

We introduce the operator $\mathcal{I}_G$ from W to $\mathcal{V}(G)$ for a given finite set $G \subset \mathbb{N}_0 \times \mathbb{N}_0^m$ by

$$\mathcal{I}_G v := \sum_{(k,\boldsymbol{s})\in G} \delta_k \Delta_{\boldsymbol{s}}^{\mathrm{I}}(v)$$

for functions v defined on $\mathbb{R}^m$ taking values in the space W .

Notice that $\mathcal{I}_G v$ is a linear non-adaptive method of fully discrete polynomial interpolation approximation which is the sum taken over the indices set G , of mixed tensor products of dyadic scale successive differences of “spatial" approximations to v , and of successive differences of their parametric Lagrange intepolating polynomials. It has been introduced in [5] (see also [6]).

Theorem 2. *Let the approximation property* (3) *hold. Let the assumptions of Lemma 1 hold for the spaces* $W^1 = V$ *and* $W^2 = W$ *for some* $0 < q_1 \le q_2 < \infty$ *with* $q_1 < 2$ *. Assume that* $(Y_n)_{n\in\mathbb{N}_0}$ *is a sequence such that every* Y_n *satisfies the condition* (4) *for some positive numbers* τ *and* C *. Define for* $\xi > 0$

$$G(\xi) := \begin{cases} \{(k,\boldsymbol{s}) \in \mathbb{N}_0 \times \mathbb{N}_0^m : 2^k\sigma_{2;\boldsymbol{s}}^{q_2} \le \xi\} & \text{if } \alpha \le 1/q_2 - 1/2; \\ \{(k,\boldsymbol{s}) \in \mathbb{N}_0 \times \mathbb{N}_0^m : \sigma_{1;\boldsymbol{s}}^{q_1} \le \xi, \; 2^{\alpha q_1 k}\sigma_{2;\boldsymbol{s}}^{q_1} \le \xi\} & \text{if } \alpha > 1/q_2 - 1/2. \end{cases}$$

Then for each $n \in \mathbb{N}$ *there exists a number* ξ_n *such that* $|G(\xi_n)| \le n$ *and*

$$\| \big(u - \mathcal{I}_{G(\xi_n)} u\big) \sqrt{g} \|_{L_\infty(\mathbb{R}^m, V)} \le C n^{-\min(\alpha,\beta)}. \tag{7}$$

The rate α *is given by* (3)*. The rate* β *is given by*

$$\beta := \left(\frac{1}{q_1} - \frac{1}{2}\right) \frac{\alpha}{\alpha + \delta}, \quad \delta := \frac{1}{q_1} - \frac{1}{q_2}.$$

The constants C *in* (7) *is independent of* u*,* m *and* n*.*

Observe that the approximant $\mathcal{I}_{G(\xi_n)} u$ in this theorem can be represented in the form of a multilevel approximation to u with $k(n)$ levels:

$$\mathcal{I}_{G(\xi_n)} u \;=\; \sum_{k=0}^{k(n)} \delta_k I_{\Lambda_k(\xi_n)} u,$$

where $k(n) := \lfloor \log_2 \xi_n \rfloor$ and for $k \in \mathbb{N}_0$ and $\xi > 0$,

$$\Lambda_k(\xi) := \begin{cases} \left\{ \boldsymbol{s} \in \mathbb{N}_0^m : \sigma_{2;\boldsymbol{s}}^{q_2} \le 2^{-k} \xi \right\} & \text{if } \alpha \le 1/q_2 - 1/2; \\ \left\{ \boldsymbol{s} \in \mathbb{N}_0^m : \sigma_{1;\boldsymbol{s}}^{q_1} \le \xi, \ \sigma_{2;\boldsymbol{s}}^{q_1} \le 2^{-k} \xi \right\} & \text{if } \alpha > 1/q_2 - 1/2. \end{cases}$$

The sequence $\left\{\Lambda_k(\xi_n)\right\}_{k=0}^{k(n)}$ is nested in the inverse order, i.e., $\Lambda_{k'}(\xi_n) \subset \Lambda_k(\xi_n)$ if $k' > k$, and $\Lambda_0(\xi_n)$ is the largest and $\Lambda_{k(n)}(\xi_n) = \{0\}$.

Further, the fully discrete polynomial interpolation approximation by operators $\mathcal{I}_{G(\xi_n)}$ is a collocation approximation based on the finite number $|\Gamma(\Lambda_0(\xi_n))| \le \sum_{\boldsymbol{s} \in \Lambda_0(\xi_n)} \prod_{j=1}^m (2s_j + 1)$ of the particular solvers $u(\boldsymbol{y})$, $\boldsymbol{y} \in \Gamma(\Lambda_0(\xi_n))$, where $\Gamma(\Lambda_0(\xi_n)) = \cup_{\boldsymbol{s} \in \Lambda_0(\xi_n)} \Gamma_{\boldsymbol{s}}$, $\Gamma_{\boldsymbol{s}} = \{\boldsymbol{y}_{\boldsymbol{s}-\boldsymbol{e};\boldsymbol{m}} : e \in \mathbb{E}_{\boldsymbol{s}};\ m_j = 0, ..., s_j - e_j,\ j \in \mathbb{N}\}$ and $\mathbb{E}_{\boldsymbol{s}}$ denotes the subset in $\mathbb{N}_0^m$ of all $\boldsymbol{e}$ such that e_j is 1 or 0 if $s_j > 0$, and e_j is 0 if $s_j = 0$, and $\boldsymbol{y}_{\boldsymbol{s};\boldsymbol{m}} := (y_{s_j;m_j})_{j \in \mathbb{N}}$.

Infinite-dimensional parametric counterparts of our problems have been studied for non-fully [7, 8] and fully [7] discrete collocation approximations. In [7], the same problem of non-fully and fully discrete polynomial interpolation approximations has been investigated. There, the difference is that $m = \infty$ and the approximation error is measured by the (quasi-)norm of $L_p(\mathbb{R}^\infty, V, \gamma)$ with $0 < p \le 2$. In particular, from [7, Corollary 5.1 and Theorem 5.2] it follows that under the hypothesis of Theorem 1 or of Theorem 2, we have $\|u - I_{\Lambda(\xi_n)} u\|_{L_p(\mathbb{R}^m, V, \gamma)} \le C n^{-(1/q_1 - 1/2)}$ or $\|u - \mathcal{I}_{G(\xi_n)} u\|_{L_p(\mathbb{R}^m, V, \gamma)} \le C n^{-\min(\alpha,\beta)}$, respectively, with the same q_1 and α, β as in (6) and (7), and constants C independent of u, m and n.

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Логнормаланған алғашқы мәліметтермен берілген параметрлі эллиптикалық дербес туындылы дифференциалдық теңдеулер үшін көпөлшемді коллокациялық салмақты жуықтаулар

Аннотация: Логнормаланған алғашқы мәліметтерімен берілген параметрлі және стохастикалық эллиптикалық дербес туындылы дифференциалдық теңдеулер үшін толық және толық емес дискретті полиномиалды интерполяциялық салмақты аппроксимацияның сызықты бейімделмеген әдістері қырылған. Осы әдістермен жуықтау жылдамдықтары алынған. Бұнда сиретілген коллокация әдістері қолданылады. Сонымен қатар, толығымен дискретті әдістерді көпдеңгейлі жуықтау әдістері деп қарастыруға болады. Смоляктың сиретілген интерполяциялық торлары параметрлі облыстарда Эрмит полиномдарының немесе олардың жетілдірілген модификацияларының түбірлерінен қырастырылған.

Түйін сөздер: көпөлшемді жуықтаулар, параметрлі және стохастикалық эллиптикалық дербес туындылы дифференциалдық теңдеулер, логнормаланған алғашқы мәліметтер, коллокация әдісі, бейімделмеген салмақты полиномиалды интерполяциялық жуықтаулар.

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Высокоразмерные коллокационные весовые приближения для параметрических эллиптических уравнений в частных производных с логнормальными входными данными

Аннотация: Разработаны линейные неадаптивные методы для неполной и полной дискретной полиномиальной интерполяции в весовых приближениях для параметрических и стохастических эллиптических уравнений в частных производных с логнормальными входными данными и установлены скорости их сходимости. Наши методы являются коллокационными методами разреженных сеток. Более того, полностью дискретные методы можно рассматривать как многоуровневые методы приближения. Разреженные интерполяционные сетки Смоляка в параметрической области построены из корней полиномов Эрмита или их улучшенных модификаций.

Ключевые слова: многомерные приближения, параметрические и стохастические элиптические уравнения в частных производных, логнормальные входные данные, метод коллокации, неаддаптивные весовые полиномиальные интерполяционные приближения.

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