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DISCRETIZATION OF SOLUTIONS OF POISSON EQUATION BY INACCURATE INFORMATION

Abstract: Partial differential equations, along with the function, derivative, and integral, are among the basic mathematical models. Their solutions, even when expressed explicitly through series or integrals, are in fact again inaccessible to direct computer calculations infinite objects. Therefore, the problem of approximating them with finite objects arises, the mathematical formulation of which is contained in the definition of the Computational (numerical) diameter.

In the article the problem of discretization by inaccurate information of solutions of Poisson equations with right-hand side f belongs to the anisotropic Korobov classes $E^{r_1,...,r_s}$ is considered. There are obtained upper bound of error of discretization by innacurate information from values at point of f in uniform metric. Wherein, the boundaries of inaccurate information which keep the order of discretization by accurate information are obtained. Computational aggregates constructed by optimal Korobov quadrature formulas with equal weights and nodes, which found by algorithms based on divisor theory.

Keywords: Poisson equation, discretization of solutions, optimal computational aggregate, inaccurate information, Computational (numerical) diameter, anisotropic Korobov classes.

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INTRODUCTION

In the article is considered the problem of discretization of solutions of Poisson equation

$$\Delta u \equiv \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + ... + \frac{\partial^2 u}{\partial x_s^2} = f(x)$$

by inaccurate information from $f \in E^{r_1,...,r_s}$ in the sense of Computational (numerical) diameter -2 problem. Let at first consider the definition of Computational (numerical) diameter problem (denoted by C(N)D).

In C(N)D the initial definition is (see, for example, [1]- [3])

$$\delta_N(\varepsilon_N; D_N)_Y \equiv \delta_N(\varepsilon_N; T; F; D_N)_Y \equiv \inf_{(l^{(N)};\varphi_N)\in D_N} \delta_N(\varepsilon_N; (l^{(N)}; \varphi_N))_Y$$

where

$$\delta_N(\varepsilon_N; (l^{(N)}; \varphi_N))_Y \equiv \delta_N(\varepsilon_N; T; F; (l^{(N)}; \varphi_N))_Y \equiv$$
$$\equiv \sup_{f\in F, |\gamma_N^{(\tau)}|\le 1(\tau=1,...,N)} \|Tf(\cdot) - \varphi_N(l_N^{(1)}(f) + \gamma_N^{(1)}\varepsilon_N^{(1)}; ..., l_N^{(N)}(f) + \gamma_N^{(N)}\varepsilon_N^{(N)}; \cdot)\|_Y.$$

Here, a *mathematical model* is given by the operator $T : F \to Y$. X and Y are the normalized spaces of functions defined on Ω_X and Ω_Y respectively, $F \subset Y$ is a class of functions. Numerical information $l^{(N)} \equiv l^{(N)}(f) = (l_N^{(1)}(f), ..., l_N^{(N)}(f))$ of volume $N (N = 1, 2, ...)$ about f from class F is taken by linear functionals $l_N^{(1)}(f), ..., l_N^{(N)}(f)$ (in general, not necessarily linear). An *information processing algorithm* $\varphi_N(z_1, ..., z_N; \cdot) : C^N \times \Omega_X \to C$ is a correspondence, which for every fixed $(z_1, ..., z_N) \in C^N$ as a function of $(\cdot)$ is an element of Y. The record $\varphi_N \in Y$ means that φ_N satisfies all the conditions listed above, and $\{\varphi_N\}_Y$ is a set composed of all $\varphi_N \in Y$. Further, $(l^{(N)}; \varphi_N)$ is a *computational aggregate* of recovery from accurate information for the function $f \in F$ acting according to the rule $\varphi_N(l_N^{(1)}, ..., l_N^{(N)}; \cdot)$.

The recovery of Tf by inaccurate information is proceeding as follows. At first the boundaries of the inaccuracy are set: a vector $\varepsilon_N = (\varepsilon_N^{(1)}, ..., \varepsilon_N^{(N)})$ with non-negative components. Then, the accurate values $l_N^{(\tau)}(f)$ are replaced with a given accuracy $\varepsilon_N^{(\tau)} \geq 0$ by approximate values $z_\tau \equiv z_\tau(f)$, $|z_\tau - l_N^{(\tau)}(f)| \leq \varepsilon_N^{(\tau)}$ ($\tau = 1, ..., N$), numbers $z_\tau \equiv z_\tau(f)$ ($\tau = 1, ..., N$) are processed using the algorithm φ_N up to the function $\varphi_N(z_1(f), ..., z_N(f); \cdot)$, which will constitute the computational aggregate $(l^{(N)}; \varphi_N) \equiv \varphi_N(z_1(f), ..., z_N(f); \cdot)$ constructed according to information of the precision $\varepsilon_N = (\varepsilon_N^{(1)}, ..., \varepsilon_N^{(N)})$.

Let $D_N \equiv D_N(F)_Y$ be a given set of complexes $(l_N^{(1)}, ..., l_N^{(N)}; \varphi_N) \equiv (l^N, \varphi_N)$, we emphasize, operators of recovery by accurate information.

Notations $A << B(B \geq 0)$ and $A \succ\prec B(A, B \geq 0)$, for $A \equiv A_n$ and $B \equiv B_n$ respectively mean $| A_n | \leq cB_n(c > 0, n = 1, 2, ...)$ and simultaneous execution $A << B$ and $B << A$.

Within the framework of given notations and definitions, the problem of optimal recovery by inaccurate information, framed under the name *Computational (numerical) diameter*, according to the [1]- [3], consists in a collective sense in sequential solution of the following three tasks: C(N)D-1, C(N)D-2 and C(N)D-3.

For given $T; F; Y; D_N$:

C(N)D-1: an order of $\succ\prec \delta_N(0; D_N)_Y \equiv \delta_N(0; T; F; D_N)_Y$ is found with the construction of a specific computional aggregate $(\bar{l}^{(N)}, \overline{\varphi}_N)$ from $D_N \equiv D_N(F)_Y$ supporting ordering

$$\succ\prec \delta_N(0; D_N)_Y;$$

C(N)D-2: for $(\bar{l}^{(N)}, \overline{\varphi}_N)$ is considered the problem of existence and finding a sequence $\widetilde{\varepsilon}_N \equiv \widetilde{\varepsilon}_N(D_N; (\bar{l}^{(N)}; \overline{\varphi}_N))_Y$ with non-negative components such that

$$\delta_N(0; D_N)_Y \equiv \delta_N(\widetilde{\varepsilon}_N; (\bar{l}^{(N)}; \overline{\varphi}_N))_Y \equiv$$

$$\equiv \sup\{\|Tf(\cdot) - \overline{\varphi}_N(z_1, ..., z_N; \cdot)\|_Y : f \in F, |\bar{l}_\tau(f) - z_\tau| \leq \widetilde{\varepsilon}_N^{(\tau)}(\tau \in \{1, ..., N\}).\}$$

with simultaneous satisfying the following expression

$$\forall \eta_N \uparrow +\infty(0 < \eta_N < \eta_{N+1}, \eta_N \to +\infty) :$$

$$\overline{\lim_{N\to+\infty}} \delta_N(\eta_N \widetilde{\varepsilon}_N; (\bar{l}^{(N)}, \overline{\varphi}_N))_Y / \delta_N(0; D_N)_Y = +\infty;$$

C(N)D-3: *massiveness* of limiting error $\widetilde{\varepsilon}_N$ is set: as huge as possible set $M_N(\bar{l}^{(N)}; \overline{\varphi}_N)$ from D_N (usually associated with the structure of the $(\bar{l}^{(N)}; \overline{\varphi}_N)$) of computional aggregates $(l^{(N)}, \varphi_N)$ is found, such that for each of them the following relation holds

$$\forall \eta_N \uparrow +\infty(0 < \eta_N < \eta_{N+1}, \eta_N \to +\infty) :$$

$$\overline{\lim_{N\to+\infty}} \delta_N(\eta_N \widetilde{\varepsilon}_N; (l^{(N)}, \varphi_N))_Y / \delta_N(0; D_N)_Y = +\infty.$$

In the article is considered the following concretization of Computational (numerical) diameter. $Tf(x) \equiv u(x, f)$ – the solution of Poisson equations $f(x) = f(x_1, ..., x_s)$

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} + ... + \frac{\partial^2 u}{\partial x_s^2} = f(x) \tag{1}$$

which satisfies zero boundary conditions on the unit cube $[0, 1]^s$. $f \in F = E^{r_1, ..., r_s}$ – anisotropic Korobov class, Y are uniform metric.

The problem of discretization by accurate information of solutions of Poisson equation whose right-hand side belongs to a Korobov class $E_s^r \equiv E^{r,...,r}$ was consideres in [4]- [8]. N. Korobov proved [4] that if $a_1, ..., a_s$ are the optimal coefficients modulo N and β index (necessary definitions see in [4]), then the following estimate holds ($N_1 = \sqrt{N} ln^{-\frac{\beta}{2}} N$):

$$\sup_{f\in E_s^r} \sup_{x\in[0,1]^s} \left| u(x,f) - \frac{1}{4\pi^2 N} \sum_{k=1}^{N} f\left(\left\{ \frac{a_1 k}{N} \right\}, ..., \left\{ \frac{a_s k}{N} \right\} \right) \sum_{\overline{m}_1 ... \overline{m}_s < N_1}^{*} \frac{e^{2\pi i \left(m_1 \left(x_1 - \frac{a_1 k}{N} \right) + ... + m_s \left(x_s - \frac{a_s k}{N} \right) \right)}}{m_1^2 + ... + m_s^2} \right| << \tag{2}$$

$$<< \frac{(lnN)^{\frac{r\beta}{2}+s}}{N^{\frac{r}{2}-\frac{1}{2}+\frac{1}{s}}},$$

here and everywhere below the asterisk over the sum symbol means that $m = (0, ..., 0)$ is dropped in the summation.

E. Bailov and N. Temirgaliyev in [5] were obtained sharp estimates in the power scale for the discretization error, which is almost doubled in the power scale in comparison with (2). In [6] by S. Kudaibergenov and S. Sabitova is obtained a sharp discretization error estimate on the power scale with the application of Smolyak grid nodes. The problem of discretization by accurate information of solutions of the Poisson equations whose right-hand side belongs to a anisotropic Korobov class $F = E^{r_1,...,r_s}$ was considered by Tashatov [7] and E. Bailov [8].

The aim of this article is discretization by innacurate information of solutions of the Poisson equations with right-hand side from the anisotropic Korobov classes $E^{r_1,...,r_s}$.

1. NECESSARY DEFINITIONS AND STATEMENTS

The anisotropic Korobov class [4] $E^{r_1,...,r_s}(r_1 > 1, ..., r_s > 1)$ consists of all 1-periodic in each variable functions $f(x) = f(x_1, ..., x_s)$, which trigonometric Fourier-Lebesgue coefficients satisfy the condition

$$|\widehat{f}(m)| = \left| \int_{[0,1]^s} f(x) e^{-2\pi i (m,x)} dx \right| \le \prod_{j=1}^{N} \overline{m}_j^{-r_j} (m = (m_1, ..., m_s) \in Z^s),$$

here and everywhere below $\overline{m}_j = max\{1, |m_j|\}$ and $(m, x) = m_1 x_1 + ... + m_s x_s$.

By renumbering the coordinates of vector $r = (r_1, ..., r_s)$ (if necessary), we will assume that the vector $r = (r_1, ..., r_s)$ has the form $r_1 = ... = r_\nu < r_{\nu+1} \le ... \le r_s$.

Set for $R \ge 1$,

$$\Gamma_R(\beta) = \{m = (m_1, ..., m_s) : m \in Z^s, \prod_{j=1}^{N} \overline{m}_j^{\beta_j} \le R\},$$

where $\beta = (\beta_1, ..., \beta_s), 1 = \beta_1 = ... = \beta_\nu < \beta_{\nu+1} \le ... \le \beta_s$.

Lemma (Y. Bailov, [8]). Let is given a vector $\gamma = (\gamma_1, ..., \gamma_s)$ such that $1 = \gamma_1 = ... = \gamma_\nu$ and $\gamma_j > 1$ for $j = \nu + 1, ..., s$. Then for any $R \ge 1$ satisfies

$$\sum_{m\in\Gamma_R(\gamma)} \frac{1}{(\overline{m}_1 \cdot ... \cdot \overline{m}_s)^\alpha} \ll \begin{cases} c(s,\alpha), \ if \ \alpha > 1, \\ ln^s R, \ if \ \alpha = 1, \\ R^{1-\alpha} ln^\nu R, \ if \ \alpha < 1. \end{cases}$$

Let F be some class of $f(x) = f(x_1, ..., x_s)$ functions, which are 1-periodic in each variable, whose trigonometric Fourier-Lebesgue series converges absolutely.

Assume that $\widehat{f}(0) \ne 0$. It is easy to verify (see, for example, [4]) that, for any boundary condition, there exists a function $\omega(x)$ depending on this condition such that $\omega(x)$ is continuous on $[0,1]^s$ and $\Delta\omega \equiv 1$ on $[0,1]^s$. Moreover, the solution of (1) has the form

$$u_\omega(x,f) = \omega(x) \cdot \widehat{f}(0) - \frac{1}{4\pi^2} \sum_{m\in Z^s}^{*} \frac{\widehat{f}(m)}{(m,m)} e^{2\pi i (m,x)} \tag{3}$$

If $f(x_1, ..., x_s)$ is odd with respect to each of its variables, then the function

$$u(x,f) = -\frac{1}{4\pi^2} \sum_{m\in Z^s}^{*} \frac{\widehat{f}(m)}{(m,m)} e^{2\pi i (m,x)} \tag{4}$$

is a solution of (1) with zero boundary conditions on $[0,1]^s$.

In [9]- [10] N. Temirgaliyev applied algebraic number theory to find the $a_1, ..., a_s$ such that the corresponding quadrature formula with the Korobov grids

$$\left\{ \left(\left\{ \frac{a_1 k}{N} \right\}, ..., \left\{ \frac{a_s k}{N} \right\} \right) \right\}_{k=1}^{N}$$

is optimal in the problem of numerical integration of functions belongs to Korobov classes E_s^r.

The algorithm for finding a prime p and integers $a_1, ..., a_s$ is as follows (necessary definitions and notations see in [9]- [10] and bibliography therein):

Let are given a prime number $l = s + 1 (3 \le l \le 19)$,

$$E = \Gamma_R(\beta) = \{m = (m_1, ..., m_s) : m \in Z^s, \prod_{j=1}^{N} \overline{m}_j^{\beta_j} \le R\}$$

where $1 = \beta_1 = ... = \beta_\nu < \beta_{\nu+1} \le ... \le \beta_s$.

1. Find the smallest prime $p, p \equiv 1(modl)$ such that does not divided $\prod_{m \in \Gamma_R(\beta)} N(m)$.

2. By searching through all integers $n_j, |n_j| \le Bp^{\frac{1}{s}} (j = 1, ..., s)$ find $n = n_1\omega_1 + ... + n_s\omega_s \in A_s(\omega_j = \theta^{j-1}, j = 1, ..., s)$ such that for the main ideal $\Re = (n)$ satisfies $N\Re = p$.

3. Write a matrix

$$d' = \begin{pmatrix} c_{11} & ... & c_{s1} \\ . & . & . \\ s_{1s} & ... & c_{ss} \end{pmatrix}$$

from the relations

$$\gamma_k = n\omega_k = (n_1 + n_2\theta + ... + n_s\theta^{s-1})\theta^{k-1} = c_{k1} + c_{k2}\theta + ... + c_{ks}\theta^{s-1}.$$

4. Write the basis $\gamma_k'' = \sum_{j=1}^{s} \nu_{kj}\omega_j (k = 1, ..., s)$ of $\Re$ with triangular matrix $(\nu_{kj})_{k,j=1}^{s}$. From the equality $p = |\nu_{11}...\nu_{ss}|$ there is a number j_0 such that $|\nu_{j_0 j_0}| = p$.

5. Write the coefficients

$$a_k = (-1)^{k+j_0} M_{j_0,k} sgndetd' (k = 1, ..., s),$$

where $M_{j_0,k}$ is additional element minor $c_{j_0,k}$ of matrix d'.

Also, everywhere below we denote by $\gamma = (\gamma_1, ..., \gamma_s)$ a vector whose components are defined by the equalities $\gamma_j = r_j / r_1 (j = 1, ..., s)$. It's clear that $1 = \gamma_1 = ... = \gamma_\nu < \gamma_{\nu+1} \le ... \le \gamma_s$.

Theorem B (Y. Bailov, [8]). Let $l = s + 1 \ \ (3 \le l \le 19)$ – prime and $r_1 > 1$. Then for any $T > c(l)$ there are exist prime $p, p \equiv 1(modl), p \le c(s)Rln^\nu R = T$ and integers $a_1, ..., a_s$, for finding which according to Algorithm 1-5, it suffices to perform $<< TlnlnT$ elementary arithmetic operations, such that

$$\sup_{f \in E^{r_1,...,r_s}} \sup_{x \in [0,1]^s} \left| u_\omega(x, f) - \frac{1}{p} \sum_{k=1}^{p} f\left(\left\{ \frac{a_1 k}{p} \right\}, ..., \left\{ \frac{a_s k}{p} \right\} \right) \cdot \right.$$

$$\left. \cdot \left(\omega(x) - \frac{1}{4\pi^2} \sum_{m \in \Gamma_{R_1}(\gamma)}{}^{*} \frac{1}{(m, m)} e^{2\pi i \sum\limits_{j=1}^{s} m_j \left(x_j - \frac{a_j k}{p} \right)} \right) \right| << b_s(T) \cdot \frac{(lnT)^{\frac{v(r_1+1)}{2r_1}(r_1 + \frac{2}{s} - 1)}}{T^{\frac{r_1}{2} - \frac{1}{2} + \frac{1}{s}}},$$

where $R_1 \asymp \sqrt{T}(lnT)^{-\frac{\nu(r_1+1)}{2r_1}}$, *and* $b_s(T)$ *is equal to* lnT *if* $s = 2$ *and to* $ln^{\nu-1}T$ *if* $s > 2$.

2. MAIN RESULTS

In the next theorem is considered odd with respect to each of its variables functions $f(x_1, ..., x_s)$.

Theorem. Let $l = s + 1 \ \ (3 \le l \le 19)$ – prime and $r_1 > 1$. Then for any $T > c(l)$ there are exist prime $p, p \equiv 1(modl), p \le c(s)Rln^\nu R = T(p) =\equiv T$ and integers $a_1, ..., a_s$, for finding which according to Algorithm 1-5, it suffices to perform $<< TlnlnT$ elementary arithmetic operations, such that for the sequence

$$\tilde{\varepsilon}_T \equiv \tilde{\varepsilon}_{T(p)} \succ\prec \begin{cases} \dfrac{(lnT)^{\frac{\nu}{2}(r_1+\frac{2}{s}-1)\frac{r_1+1}{r_1}-(s-1)}}{T^{\frac{r_1}{2}+\frac{1}{s}-\frac{1}{2}}}, if s=2, \\ \dfrac{(lnT)^{\frac{\nu}{2}(r_1+1)-1}}{T^{\frac{r_1}{2}}}, if s>2 \end{cases}$$

satisfyes

$$\sup_{f\in E^{r_1,\dots,r_s}} \sup_{x\in[0,1]^s, \left|\gamma_p^{(k)}\right|\le 1(k=1,\dots,p)} \left| u(x,f) - \frac{1}{p}\sum_{k=1}^{p}\left(f\left(\left\{\frac{a_1k}{p}\right\},\dots,\left\{\frac{a_sk}{p}\right\}\right) + \tilde{\varepsilon}_T\gamma_p^{(k)}\right)\cdot\right.$$

$$\left.\cdot\left(-\frac{1}{4\pi^2}\sum_{m\in\Gamma_{R_1}(\gamma)}{}^{*}\frac{1}{(m,m)}e^{2\pi i\sum_{j=1}^{s}m_j\left(x_j-\frac{a_jk}{p}\right)}\right)\right| << b_s(T)\cdot\frac{(lnT)^{\frac{v(r_1+1)}{2r_1}(r_1+\frac{2}{s}-1)}}{T^{\frac{r_1}{2}-\frac{1}{2}+\frac{1}{s}}},$$

where $R_1 \succ\prec \sqrt{T}(lnT)^{-\frac{\nu(r_1+1)}{2r_1}}$ *, and* $b_s(T)$ *is equal to* lnT *if* $s=2$ *and to* $ln^{\nu-1}T$ *if* $s>2$.

CONCLUSION

In the article is obtained upper bound of error of discretization by inaccurate informations of solutions of Poisson equations with right-hand side $f(x_1,\dots,x_s)\in E^{r_1,\dots,r_s}$ is odd with respect to each of its variables functions. Theorem shows that if in computational aggregates accurate numerical informations $f\left(\left\{\frac{a_1k}{p}\right\},\dots,\left\{\frac{a_sk}{p}\right\}\right)$ replaced with innacurate informations $f\left(\left\{\frac{a_1k}{p}\right\},\dots,\left\{\frac{a_sk}{p}\right\}\right)+\tilde{\varepsilon}_T\gamma_p^{(k)}\left(\left|\gamma_p^{(k)}\right|\le 1(k=1,2,\dots,p)\right)$ then the order of error of approximation by accurate information is keeping.

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Дәл емес ақпарат бойынша Пуассон теңдеуінің шешімдерін дискретизациялау

Аннотация: Дербес туындылы дифференциалдық теңдеу шешімдері функция, туынды, интегралмен қатар негізі математикалық объектілер қатарына жатады. Олардың шешімдері, тіпті айқын түрде қатар және интегралдар арқылы берілген жағдайлардың өзінде, тікелей компьютерлік есептеулер жүргізуге мүмкін емес шексіз объектілер болып табылады. Сол себепті, ақырлы объектілермен жуықтау есебі туындайды, бұл есептің математикалық қойылымы Компьютерлік (есептеуіш) диаметр анықтамасына енеді.

Мақалада оң жағы f анизотропты Коробов классында $E^{r_1,\ldots,r_s}$ жататын Пуассон теңдеуінің шешімдерін дәл емес мәлімет бойынша жуықтау қарастырылады. Бірқалыпты метрикада f функциясының нүктедегі дәл емес мәндері бойынша шешімін жуықтау қателігінің жоғарыдан бағалауы алынды. Дәл мәлімет бойынша жуықтау қателігін сақтайтын дәл емес ақпараттың шекаралары анықталды. Мақаладағы есептеу агрегаттары Коробовтың бірдей салмақты және дивизорлар теориясына негізделген алгоритмдерді қолданып есептелген торлар арқылы құрылған.

Түйін сөздер: Пуассон теңдеуі, шешімдерді дискретизациялау, оптималды есептеу агрегаты, дәл емес ақпарат, Компьютерлік (есептеуіш) диаметр, анизотропты Коробов класстары.

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Дискретизация решений уравнений Пуассона по неточной информации

Аннотация: Уравнения в частных производных наряду с функцией, производной, интегралом относятся к основным математическим моделям. Их решения, даже в случае явного выражения посредством рядов или интегралов, фактически опять же представляют собой недоступные к прямым компьютерным вычислениям бесконечные объекты. Здесь снова возникает задача приближения конечными объектами, математическая формулировка которой содержится в определении Компьютерного (вычислительного) поперечника.

В статье изучается задача дискретизации решений уравнения Пуассона с правой частью f из анизотропных классов Коробова $E^{r_1,\ldots,r_s}$ по неточной информации. Получены оценки сверху погрешности в равномерной метрике дискретизации по информации, составляющих значения функции f в точках, вычисленных с ошибкой. При этом указаны границы неточности информации, сохраняющие порядки убывания погрешности восстановления, вычисленных по точной информации. Вычислительные агрегаты построены по оптимальным квадратурным формулам Коробова с равными весами и узлами, основанным на теории дивизоров алгоритмам.

Ключевые слова: Уравнение Пуассона, дискретизация решений, оптимальный вычислительный агрегат, неточная информация, Компьютерный (вычислительный) поперечник, анизотропные классы Коробова.

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